

A loudspeaker-driven clarinet for educational purpose

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Abstract – This paper describes a toy device, namely an electroacoustic clarinet, which was designed for educational purposes. It consists of a loudspeaker connected through a cavity to a duct drilled with several side holes. Unlike what happens in a clarinet where the emitted sound is caused by the motion of a reed and the blowing of a musician, the present system is excited by the loudspeaker through a positive feedback mechanism giving rise to self-oscillations. This feedback is achieved by placing a microphone in the cavity, and connecting it to the loudspeaker through an audio amplifier. It is shown that, by adjusting the gain of the amplifier and by opening/closing the side holes to control the pitch of the emitted sound, the system can spontaneously play all the notes of a scale. The dynamics of the system beyond the threshold of self-oscillations is also studied both experimentally and theoretically. This simple system can be easily reproduced with minimum equipment and it may have some merits for educational purpose, because its understanding requires some basic knowledge of electroacoustics, guided wave theory and nonlinear dynamics.

Keywords: Musical acoustics, Nonlinear dynamics

1 Introduction

The physics of musical instruments is an endless topic of research for acousticians. The basic principles of musical instruments are nowadays well understood and documented [1, 2], but there is always room for (and pleasure in) studying further their physics. Moreover, musical instruments often share some features with other systems or other engineering issues, which involves making use of similar approaches for understanding the physics of sound radiation, the dynamics of a system, the role of nonlinear effects or the coupling between resonators, for instance. Therefore, the study of musical instruments is also an interesting way of stimulating students interest and tackling concepts and tools that are useful elsewhere.

In this paper, we investigate both theoretically and experimentally the dynamics of a system that might be classified as a wind instrument, although it is neither a brass nor a woodwind instrument. In this system, the self-oscillations of an air column are excited by means of a moving coil loudspeaker connected through an amplifier to a microphone facing the gas column. Self-oscillations are generated above a threshold value of the gain in the feedback loop, and the pitch of the sound emitted is controlled by opening or closing side holes along the air column. The description of this “electroacoustic clarinet” proposed below is mostly motivated by the fact that it is an interesting

system for teaching purposes, because its study by a bachelor or a master student in acoustics can call up a wide range of skills.

The experimental set-up is described in Section 2, and a linear model enabling to predict the threshold of self-oscillations and the fundamental frequency of the sound emitted is proposed in Section 3. The experimental study of the system is presented in Section 4, where the amplitude and frequency content of self-oscillations are studied as functions of the applied fingering and of the gain of the feedback loop. In Section 5, a nonlinear model of the system is presented which accounts for the saturation of sound caused by the feedback microphone. The dynamics of the system is then studied numerically and compared with experiments. Some concluding remarks are drawn in Section 6. Finally, additional information regarding the description of the acoustic load, some calculation details for the nonlinear model and additional experiments are provided in the appendices.

2 Experimental set-up

A schematic representation and some photographs of the system considered are presented in Figure 1. A moving coil loudspeaker (model Aura Sound NSW1-205-8A) is attached to the left side of a PVC cylinder (inner diameter of 34 mm) of length L_{cav} which is used as a coupling cavity between the loudspeaker and a long duct. This duct has a

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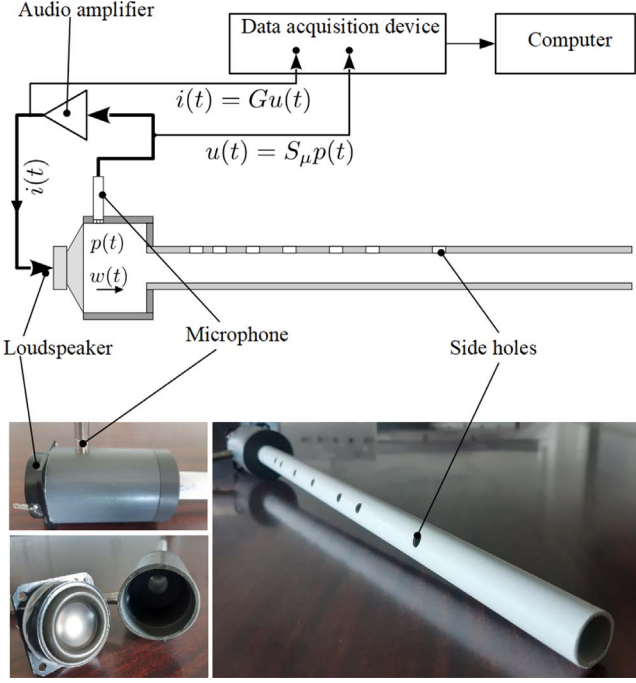


Figure 1. Sketch and photograph of the experimental set-up.

length L_d and an inner diameter of 13.1 mm, and it is drilled with several side holes (5 mm in diameter). A standard 1/4" measurement microphone (model ROGA MI-19, sensitivity $S_\mu = 48 \text{ mV} \cdot \text{Pa}^{-1}$, max SPL 130 dB) is mounted flush in the cavity, very close to the loudspeaker's membrane. This microphone is part of the feedback loop, which consists of a custom-made current-drive amplifier [3, 4] delivering to the loudspeaker a current $i(t)$ proportional through an adjustable gain G to the acoustic pressure $p(t)$ sensed by the microphone.

As will be shown in the following, there exists a threshold value of the gain G above which self-sustained acoustic oscillations are spontaneously emitted by the system, thanks to the controlled audio feedback. Moreover, the frequency of acoustic oscillations can be tuned by opening or closing some of the side holes, as is the case for a clarinet.

3 Linear model and marginal stability

3.1 Linear model

A simple and very common way of describing a moving coil loudspeaker amounts to considering that it is a mass-spring-damper system driven by a force proportional to the electrical current passing through the coil. More precisely, considering that an oscillating current $i(t)$ is applied to the loudspeaker, this gives rise to an oscillating force $B\ell i(t)$ along the loudspeaker axis, such that the displacement $x(t)$ of the membrane can be obtained by writing the second Newton's law as follows:

$$M_m \ddot{x} + R_m \dot{x} + K_m x = B\ell i - pS_m \quad (1)$$

where M_m stands for the mass of the membrane, where R_m and K_m denote the mechanical resistance and the stiffness

of the suspensions, and where $B\ell$ is the so-called force factor of the loudspeaker [5–7]. The force $-pS_m$ results from the reaction of the acoustic load (namely the cavity and the duct), where $p(t)$ denotes the pressure in the cavity and S_m is area of the membrane. Note that the pressure $p(t)$ facing the membrane in the cavity is assumed to be uniform. In the frequency range of interest, non planar modes are indeed evanescent (the first non plane wave mode has a cut-off frequency of about 6 kHz). Note also that in the above equation the force of reaction due to radiation of sound by the rear side of the loudspeaker is neglected (or in other words, the pressure facing the rear side of the membrane is much lower than the pressure $p(t)$ in the cavity, so it does not affect the motion of the membrane).

The electroacoustic feedback loop gives a relation between the pressure $p(t)$ measured by the microphone and the current $i(t)$ supplied to the loudspeaker as follows:

$$i = Gu = GS_\mu p, \quad (2)$$

where $u(t) = S_\mu p(t)$ stands for the voltage delivered by the microphone, S_μ is the microphone sensitivity and G stands for the gain of the audio amplifier.

The pressure in front of the loudspeaker can be related to the displacement of the membrane by describing sound propagation through the cavity and the duct. Assuming linear propagation of plane waves through the resonator, and making use of a formulation in the frequency domain, it is possible to get a simple relation as:

$$\tilde{p} = Z_{\text{load}} \tilde{w}$$

where $\tilde{p}$ and $\tilde{w}$ stand for the complex amplitudes of the pressure in the cavity and the volume velocity of the membrane, respectively. The acoustic impedance Z_{load} can be calculated from the dimensions of the cavity and the duct as well as the size and positions of side holes, using conventional models based on transfer matrix formulation[8]. Here we use a $+i\omega t$ temporal convention, such that $p(t) = \Re(\tilde{p}e^{i\omega t})$ for a sinusoidal signal where ω is the angular frequency of acoustic oscillations. As the volume velocity of the membrane is directly related to its displacement, i.e. $w = S_m \dot{x}$, the equations above can be combined as follows:

$$\frac{Z_m}{S_m^2} \tilde{w} = \left(\frac{B\ell S_\mu G}{S_m} - 1 \right) Z_{\text{load}} \tilde{w} \quad (3)$$

where $Z_m = i\omega M_m + R_m + K_m/(i\omega)$ is the mechanical impedance of the loudspeaker. The parameters M_m , R_m , K_m and $B\ell$ can be obtained using standard methods of loudspeaker characterization based on the measurement of its electrical impedance as a function of the driving frequency [7]. The measured values of those electromechanical parameters are given in Table 1.

3.2 Marginal stability condition

Introducing the parameter $\Gamma_0 = B\ell S_\mu G/S_m$ as the global effective gain of the feedback loop, and $Z_{\text{ma}} = Z_m/S_m^2$ as the acoustical impedance of the loudspeaker, equation (3) can be rewritten as:

Table 1. Parameters of the loudspeaker.

Parameter	Measured value
M_m	$0.55 \cdot 10^{-3} \cdot \text{kg}$
R_m	$0.39 \text{ kg} \cdot \text{s}^{-1}$
K_m	$1755 \text{ N} \cdot \text{m}^{-1}$
$B\ell$	$1.92 \text{ T} \cdot \text{m}$
S_m	$5.3 \cdot 10^{-4} \cdot \text{m}^2$

$$[Z_{ma} + (1 - \Gamma_0)Z_{load}]\tilde{w} = 0. \quad (4)$$

The impedance Z_{load} can be calculated using a transfer matrix model, which is presented in [Appendix A](#). Equation (4) must be satisfied and, if the trivial solution $\tilde{w} = 0$ is discarded, there must be (at least) an angular frequency ω such that $Z_{sum}(\omega) = 0$, where $Z_{sum} = Z_{ma} + (1 - \Gamma_0)Z_{load}$. For an arbitrary choice of the gain Γ_0 , one must seek for some complex root $\omega = \omega_r - 2i\pi\sigma$, where the real part ω_r corresponds to the natural frequency of acoustic oscillations, and where σ stands for the corresponding growth rate. The marginal stability condition is achieved when the free oscillations of the system are neither damped ($\sigma < 0$) nor amplified ($\sigma > 0$). Therefore, by solving numerically $Z_{sum}(\omega) = 0$ for different values of the feedback gain Γ_0 , it is possible to determine the threshold value of the gain such that $\sigma = 0$, which corresponds to the onset of self-oscillations at angular frequency ω_r . Note that the onset necessarily occurs for $\Gamma_0 > 1$ because both Z_{ma} and Z_{load} have a positive real part (due to losses by dissipation and radiation).

The preceding comments are illustrated in [Figure 2](#) for the example of a load which is made up of a cavity with a length of 2 cm, connected to a duct with a length of 30 cm. The moduli and phases of Z_{ma} and Z_{load} are plotted as functions of the frequency in [Figure 2a](#), together with the modulus of $Z_{sum} = Z_{ma} + (1 - \Gamma_0)Z_{load}$ which is plotted both for $\Gamma_0 = 1.04$ and for $\Gamma_0 = 4.5$. As expected, the mechanical impedance Z_{ma} reaches a minimum when $\omega \approx \sqrt{K_m/M_m}$ while the acoustic load Z_{load} has several peaks and anti-peaks. For a feedback gain $\Gamma_0 = 1.04$, the combined impedance Z_{sum} has a sharp minimum around 200 Hz, which is close to the first maximum of Z_{load} . This suggests that for that value of Γ_0 , the system is prone to self-oscillate at 200 Hz. The same conclusion can be drawn for a larger value of $\Gamma_0 = 4.5$ where a higher order mode of acoustic oscillations seems to be triggered at a frequency of about 600 Hz. Those qualitative observations from a graphical analysis can be confirmed more rigorously by solving equation (4), which is done in [Figure 2b](#). A Newton-Raphson method is employed to find the roots $\omega = \omega_r - 2i\pi\sigma$ of $Z_{sum} = 0$ as functions of the feedback gain Γ_0 . The root corresponding to the first mode is plotted with red lines, while that of the second mode is plotted with blue lines. The results confirm that the onset condition for the first mode is reached for $\Gamma_0 \approx 1.045$ and that the corresponding frequency of oscillations, $f_r = \omega_r/(2\pi)$, is of about 202 Hz. The second mode becomes unstable for a much larger value of $\Gamma_0 \approx 4.52$ with the corresponding

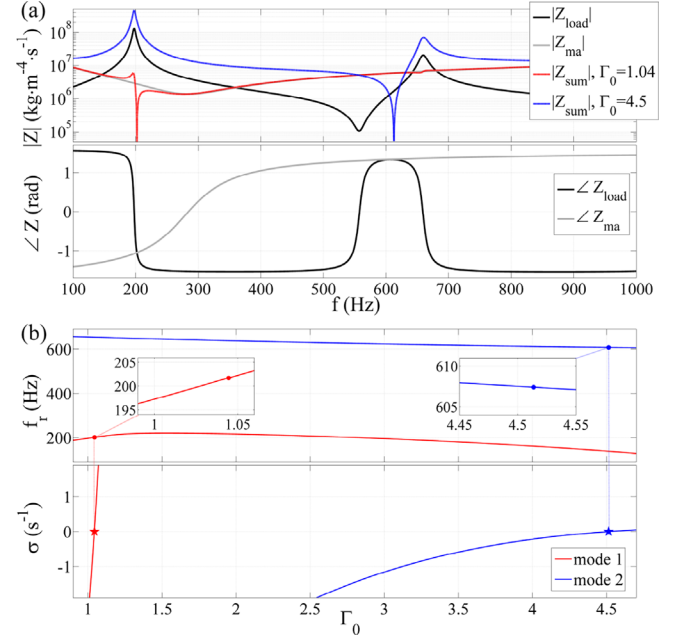


Figure 2. Analysis of marginal stability conditions for $L_{cav} = 2$ cm, and for a duct of length $L_d = 30$ cm without side holes. (a) The modulus and phase of Z_{load} (black lines) and Z_{ma} (grey lines) are plotted as functions of the frequency. The modulus of $Z_{sum} = Z_{ma} + (1 - \Gamma_0)Z_{load}$ is also plotted as a function of the frequency, either for $\Gamma_0 = 1.02$ (red line) or for $\Gamma_0 = 2$ (blue line). (b) The frequency f_r and the growth rate σ are plotted as functions of the feedback gain Γ_0 for the first (red line) and the second (blue line) modes of the system.

frequency $f_r \approx 607$ Hz. From a more practical standpoint, this means that for moderate values of the feedback gain, only the lowest order mode of oscillations is prone to self-oscillate.

Next, the possibility to change the frequency of self-oscillations by changing the length of the duct, or by opening side-holes, is analyzed. The marginal stability conditions are obtained by solving $Z_{sum} = 0$ and increasing the gain Γ_0 until $\sigma = 0$. This operation is repeated either by changing the length of the duct, or by placing open side holes along the duct. The results obtained are plotted in [Figure 3](#). For the case without side holes, the results show that as the length of the duct is shortened the frequency of self-oscillations increases, and that the gain needed to trigger self-oscillations depends on the length of the duct (a minimum occurs when the frequency of self-oscillations is around the mechanical resonance of the loudspeaker). For the case of a duct with a fixed length and several side holes, the results show that the frequency increases when the distance between the cavity and the first open hole decreases. Such a result is also expected, because as a first approach a side hole mostly acts as an open termination, such that opening a side hole is almost the same as shortening the duct. Here, the positions of the holes are set such that the eight notes of a musical scale can be played, from a G3 up to a G4. Hence, the next step is to check from experiments that the system operates as predicted.

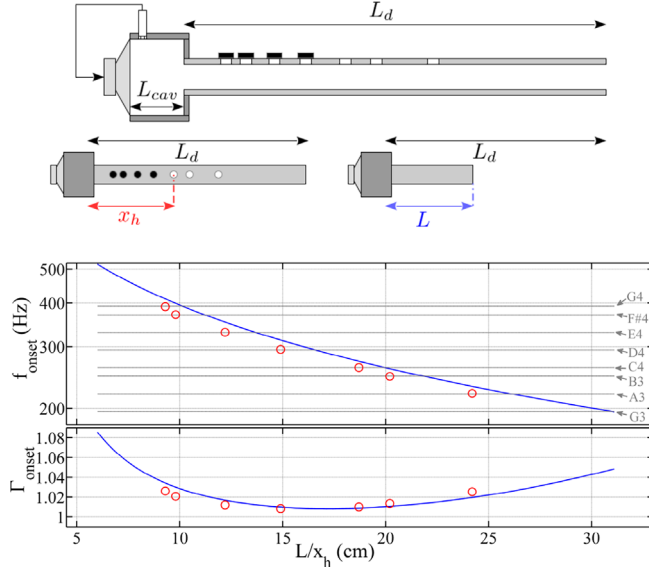


Figure 3. Onset conditions for a duct with varying length, or with side holes. For the case without side holes, the onset frequency f_{onset} and the corresponding feedback gain Γ_{onset} are plotted as functions of the length L , with solid blue lines. For the case with side holes (red markers), the length of the duct is fixed to $L_d = 31.1$ cm, the holes (5 mm in diameter) are left open one by one starting from the open termination, and the corresponding onset frequency and feedback gain are plotted for each fingering. The positions of the holes are set such that the 8 notes of a musical scale can be played, from a G2 up to a G3.

4 Experimental results

Rather than using the model presented above, and for some reasons that will be explained in the following, the length L_d of the duct as well as the positions x_h of the side holes were determined empirically after several trial and errors. The final dimensions of the duct drilled with side holes are reported in Table 2. This duct was connected to the remaining part of the system, and several tests were performed by varying either the length of the cavity L_{cav} , the value of the feedback gain Γ_0 , or the positions of open side holes.

4.1 Steady-state oscillations just above onset

In Figure 4, some experiments performed for a feedback gain $\Gamma_0 \approx 1.5$ are reported. Such a value corresponds in practice to the lowest gain giving rise to self-oscillations when all the side holes are kept close. For this value of Γ_0 , self-oscillations were also triggered for any other fingering. As it was not easy in practice to set the feedback gain accurately by just turning the potentiometer of the audio amplifier as delicately as possible, we did not try to determine experimentally the threshold value of the gain as a function of the position of the open side holes. It is also worth noting that the gain Γ_0 is not known very accurately, since it is calculated from the measurement of the pressure

Table 2. Position of the side holes, with respect to the connection between the duct and the cavity. The holes have a diameter of 5 mm. The duct has a length $L_d = 352$ mm and an inner diameter of 13.1 mm.

Hole number	x_h	Hole number	x_h
# 1	68 mm	# 5	193 mm
# 2	85 mm	# 6	217 mm
# 3	117 mm	# 7	272 mm
# 4	152 mm		

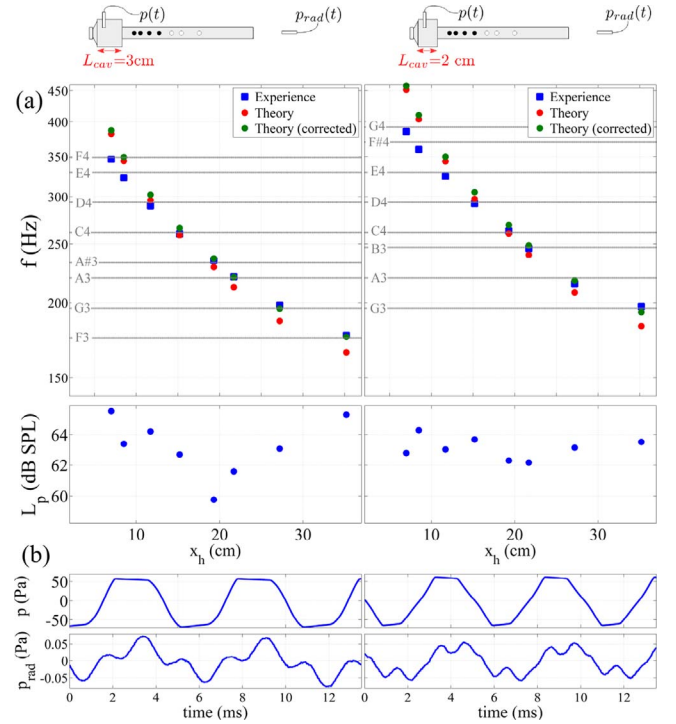


Figure 4. (a) Measured fundamental frequency (blue markers) and sound pressure level L_p of the sound emitted 20 cm away from the open end of the duct as functions of the applied fingering, for a feedback gain $\Gamma_0 \approx 1.5$, for both cases of $L_{\text{cav}} = 3$ cm and $L_{\text{cav}} = 2$ cm. The predicted onset frequencies of self-oscillations are also reported with red markers, as well as those for a modified model which accounts for a high-pass filtering of the feedback with a cut-off frequency of 8 Hz (green markers). (b) The corresponding acoustic pressures for both cases of $L_{\text{cav}} = 3$ cm and $L_{\text{cav}} = 2$ cm and when all side holes are kept closed. $p(t)$ and $p_{\text{rad}}(t)$ stand for the pressure in the cavity, and that 20 cm away from the open end of the duct, respectively.

$p(t)$ and the current $I(t)$, but it also depends on S_m , $B\ell$ and S_μ which themselves are only known with an accuracy of about 5–10%.

As shown by the blue markers in Figure 4a, the selected positions of the side holes enable to play roughly the eight notes of a musical scale, with a tuning accuracy that could probably be improved by slightly changing the position of the holes. Interestingly, the results also show that by changing the length of the cavity L_{cav} , the system can play

either from a F3 to a F4 (with $L_{\text{cav}} = 3$ cm) or from a G3 to a G4 (with $L_{\text{cav}} = 2$ cm).

The predicted onset frequencies derived from the linear model are also reported in Figure 4a, with red markers. The results show that the model is not in agreement with measurements. There are several explanations for such a disagreement. Firstly, the experimental results cannot be directly compared to the model, since the acoustic pressure is measured above threshold, once oscillations have been triggered and have reached steady-state to a finite amplitude controlled by nonlinear saturating processes. The latter nonlinear processes may influence the fundamental frequency of self-oscillations, as will be shown in the following. Secondly, the model described in Section 3 may not capture all the details of the real system. In particular, a plausible defect of the model could be due to the fact that in the real system, the feedback transfer function slightly depends on frequency, especially at very low frequency. The frequency response function of the amplifier was indeed measured, and we found that it is a first order high-pass filter with a cut-off frequency of 2.9 Hz. Moreover, measurement microphones are usually equipped with a static pressure equalization vent which also acts as a high-pass filter, and we found from the datasheet of the microphone supplier that there exists a low cut-off frequency around 10 Hz. This high-pass filtering by the feedback loop can be taken into account in the model by replacing the constant gain Γ_0 in equation (4) with the following transfer function

$$\Gamma(\omega) = \Gamma_0 \frac{i\omega/\omega_0}{1 + i\omega/\omega_0} \quad (5)$$

where ω_0 is the cut-off angular frequency. In Figure 4a, the resulting corrected predictions of the onset frequency are presented with green markers, where a cut-off frequency of 8 Hz was chosen. The results show that a better agreement is achieved, although the cut-off frequency of the feedback loop is far below the expected frequencies of self-oscillations in the present system. Therefore, even a very small phase shift due to the feedback loop (here, about 0.04 rad at 200 Hz) strongly impacts the frequency of sound emission.

In Figure 4b, the steady-state acoustic pressure $p(t)$ measured by the feedback microphone is plotted as a function of time for both cases of $L_{\text{cav}} = 3$ cm and $L_{\text{cav}} = 2$ cm and when all side holes are kept closed. The results show a saturation of the signal sensed by the microphone. The signal indeed distorts at some amplitude which does not exceed a few tens of Pa, and considering the available information on the maximum pressure that can be supported by the microphone (130 dB SPL), this clearly indicates that this signal does not correspond to the actual pressure in the cavity $p(t)$ but rather to the one sensed by the microphone, $u(t)/S_\mu$. As the voltage $u(t)$ is fed back into the loudspeaker in this system, this means that the microphone itself is a source of sound saturation. The pressure $p_{\text{rad}}(t)$ measured by another microphone placed 20 cm away from the open end of the duct is also plotted in Figure 4b. The pressure radiated outside the duct has a much lower amplitude than the one in the cavity, as expected, and

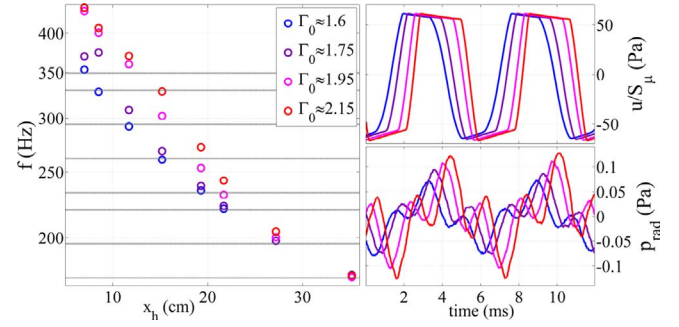


Figure 5. Measured fundamental frequency of self-oscillations as a function of the applied fingering, for different values of the feedback gain Γ_0 , and for $L_{\text{cav}} = 3$ cm. The acoustic pressure sensed by the feedback microphone, $u(t)/S_\mu$, and that emitted 20 cm away from the open end of the duct, $p_{\text{rad}}(t)$, are also plotted for the case of all side holes being kept closed.

the signal is not harmonic (in particular a third harmonic component is clearly visible).

4.2 Influence of the feedback gain

Further experiments were performed with the same device but for various amplitudes of the feedback gain Γ_0 . The results are presented in Figure 5, for the case of a cavity of length $L_{\text{cav}} = 3$ cm. The results show that, when all the side holes are closed, the value of Γ_0 only has a weak impact on the frequency of the emitted sound. However, when the targeted pitch is higher, i.e. with open side holes, the gain Γ_0 strongly impacts the frequency of self-oscillations. This can be seen for the case of all side holes being left open, where the fundamental frequency is of about 350 Hz for $\Gamma_0 \approx 1.6$, but grows up to about 440 Hz for $\Gamma_0 \approx 2.15$. Therefore, there exists some nonlinear processes which are not only responsible for the saturation of wave amplitude growth, but also impact the frequency of self-oscillations. Some time-varying signals for the measured pressure $u(t)/S_\mu$ in the cavity and the radiated pressure $p_{\text{rad}}(t)$ are also presented in Figure 5, for the case of all side holes being closed (a case for which the frequency of self-oscillations does not depend on Γ_0). Again, the signal from the feedback microphone remains limited to a few tens of Pa, whatever the applied gain of the feedback loop. The results also show that the amplitude of the radiated pressure grows when the gain increases, since the root-mean-square amplitude of the radiated pressure is about 0.04 Pa for $\Gamma_0 \approx 1.6$, and it is about 0.06 Pa for $\Gamma_0 \approx 2.15$. As was also shown in Figure 4b, there exists a visible contribution of the third harmonic in the temporal signal of the radiated pressure, and this contribution increases with an increasing gain: applying a Fourier series decomposition to the time signal, we found that the amplitude of third harmonic of p_{rad} is about 55% of the fundamental when $\Gamma_0 \approx 1.6$, whereas it grows up to 102% when $\Gamma_0 \approx 2.15$. Finally, the results presented in Figures 4 and 5 show that a nonlinear model would be needed to predict the amplitude and the spectral content of steady-state oscillations.

5 Nonlinear model

A first step in the development of a nonlinear model is to identify the sources of sound saturation. The results of Figures 4 and 5 clearly indicate that the leading source of sound saturation in the present system is due to the feedback microphone. The steady-state amplitude of the acoustic pressure measured in the cavity indeed seems consistent with the maximum pressure level that can be supported by the microphone, according to the supplier's information. Moreover, we could confirm experimentally this claim, and this was done by replacing the present microphone with another one supporting much higher sound pressure levels. We found that the emitted sound is much louder in this latter case, as shown in Appendix C.

Still, other processes might also be responsible for some nonlinear losses (but they will be ignored based on the arguments below). One is the generation of vorticity next to geometrical singularities, such as at the connection between the cavity and the duct, next to the side holes, or at the open end of the duct. Another one could be due to the loudspeaker which has a limited excursion of ± 4.5 mm, and should be described as a nonlinear oscillator for such a range of displacement. The potential role of the two above mentioned processes can be evaluated roughly from the evaluation of the amplitude of the steady-state acoustic pressure in the cavity, which does not (seem to) exceed 70 Pa in experiments. Considering the case of a duct with closed side holes, we know that the frequency of self-oscillation is about 175 Hz. From the knowledge of Z_{load} at that frequency, it is possible to evaluate a peak amplitude of about $1.4 \cdot 10^{-4} \text{ m}^3 \cdot \text{s}^{-1}$ for the volume velocity. This volume velocity is that provided by the loudspeaker's membrane with a known cross-sectional area S_m , such that the corresponding velocity of the membrane does not exceed $8 \text{ mm} \cdot \text{s}^{-1}$ and its peak displacement does not exceed $10 \mu\text{m}$, which is much less than the maximum displacement of 4.5 mm. Also, by describing sound propagation through the cavity and the duct at 175 Hz, the peak amplitude of the velocity at the open end of the duct is estimated to about $0.8 \text{ m} \cdot \text{s}^{-1}$. For such a value of the velocity, the effects of vorticity generation are not expected to lead to a large amount of nonlinear losses [9–11]. Therefore, attention is focused in the following on a simplified description of the saturation caused by the feedback microphone.

5.1 Description of the nonlinearity of the microphone

The nonlinear response of the microphone was characterized experimentally as described in the following. Firstly, the feedback loop was removed from the system, and the amplifier input was connected to a function generator instead. The ratio of the acoustic pressure in the cavity to the current supplied to the loudspeaker, $\tilde{p}/\tilde{i}$, was then measured as a function of frequency, and several resonance peaks were found. Secondly, the frequency was set to that of the first resonance peak (124 Hz), and the signal $u(t)$ delivered by the microphone was measured for a gradually increasing amplitude of the current supplied to the loudspeaker.

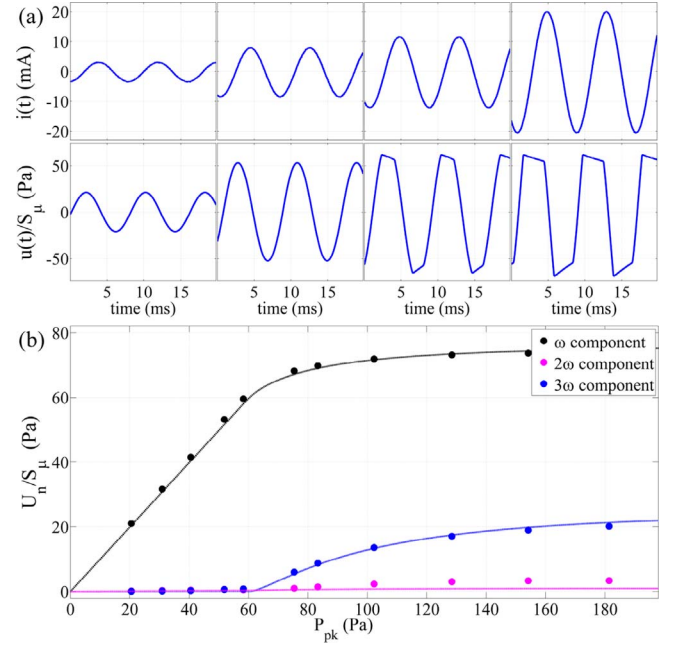


Figure 6. (a) The current $i(t)$ and the acoustic pressure $u(t)/S_\mu$ sensed by the microphone as functions of time, for various amplitudes of the assigned current (the frequency is 124 Hz). (b) Fundamental and higher harmonics components of the apparent pressure, $u(t)/S_\mu$, as functions of the peak amplitude P_{pk} of the actual pressure in the cavity $p(t)$ (known from $i(t)$). Experimental data are shown with filled markers, and theoretical data calculated from equation (6) are plotted with dashed lines.

The results are shown in Figure 6a and the distortion of the microphone signal is clearly observed when the peak amplitude of the current delivered to the loudspeaker exceeds a value of about 10 mA (which correspond to about 60 Pa for the peak amplitude of acoustic pressure in the cavity). Below this threshold value the microphone signal is monochromatic and its peak amplitude is proportional (through its sensitivity S_μ) to the peak amplitude P_{pk} of acoustic pressure in the cavity. However, above this threshold value the microphone signal is polychromatic and can be written in terms of a Fourier series expansion as $u(t) = U_1 \cos(\omega t + \phi_1) + U_2 \cos(2\omega t + \phi_2) + \dots$. In Figure 6b, the measured amplitudes of the first, second and third harmonic components of the microphone signal are presented with filled markers as functions of the peak amplitude of the acoustic pressure in the cavity.

The experimental results clearly show that when the peak amplitude of acoustic pressure in the cavity exceeds a value of about 60 Pa, the signal delivered by the microphone deviates from a monochromatic signal, since the amplitude of the first harmonic no longer grows linearly while second and (mostly) third harmonic components are generated as the acoustic pressure increases.

From the measured nonlinear response of the microphone, a function relating the microphone signal $u(t)$ to the acoustic pressure $p(t)$ can be proposed, with the goal of reproducing the results of Figure 6b. The following function is proposed

$$\frac{u(t)}{S_\mu} = \frac{1}{2} \log \left(\frac{\cosh(p(t) + p_{\max})}{\cosh(p(t) - p_{\max})} \right), \quad (6)$$

where $p(t)$ is expressed in Pa, and where the value of $p_{\max}$ is fixed to 60 Pa. By assuming that $p(t)$ is a monochromatic signal of amplitude P_{pk} , we calculated the fundamental and higher harmonics components of $u(t)$ as defined in equation (6). Those amplitudes are shown in Figure 6b with dashed lines, and it is found that the proposed describing function [12] reproduces satisfactorily the saturation and the higher harmonics generation of the microphone signal.

5.2 Nonlinear model

A time domain formulation is needed to describe the dynamics of the system by means of a set of ordinary differential equations. Therefore the description of the acoustic load in the frequency domain in terms of an impedance Z_{load} needs to be transcribed in the time domain, in terms of a linear operator. This is done by representing the load impedance as a sum of real modes in the following form:

$$Z_{\text{load}} \approx \sum_{n=1}^N \frac{i\omega A_n}{\omega_n^2 + 2i\xi_n\omega_n\omega + (i\omega)^2}. \quad (7)$$

As a result, the relation between the pressure in the cavity and the volume velocity of the membrane, which writes as $\tilde{p} = Z_{\text{load}}\tilde{w}$ can be easily written in the time domain in the following form:

$$\ddot{p}_n + 2\xi_n\omega_n\dot{p}_n + \omega_n^2 p_n = A_n\dot{w} \quad (8)$$

with

$$p(t) = \sum_{n=1}^N p_n(t). \quad (9)$$

To obtain the modal parameters ω_n , ξ_n and A_n , a peak-picking algorithm developed by Ablitzer [13] is used. In Figure 7, the modulus of Z_{load} calculated from the exact model described in Appendix A is plotted with solid blue lines as a function of frequency, and for different fingerings. The peak-picking algorithm is applied to all those load impedances, and the resulting fitted impedances are also plotted in Figure 7 with magenta solid lines. The results show that the approximated versions of Z_{load} match almost perfectly with the calculated ones. Note that depending on the fingering, a number of $N = 4$ up to $N = 6$ modes is needed for a satisfactory fitting.

Finally, by combining the equations describing the coupling between the loudspeaker and the load, equations (1) and (3), those describing the acoustic load in the time domain, equations (7)–(9), the ones describing the feedback loop, equations (2) and (5), and the one describing the saturation by the microphone, equation (6), the following equation is obtained:

$$M_{\text{ma}}\ddot{w} + R_{\text{ma}}\dot{w} + K_{\text{ma}}w + p - \frac{d_u^2}{\omega_0^2 + d_t}\Gamma(p) = 0 \quad (10)$$

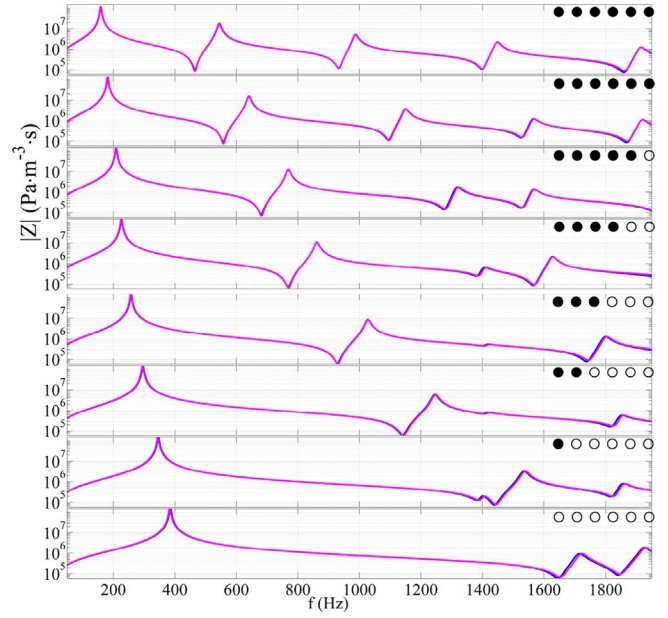


Figure 7. Comparison between the calculated (solid blue line) and the fitted (magenta line) moduli of the load impedance for different fingerings, from all side holes closed on top (●●●●●●) to all side holes open. The hole positions are those given in Table 2.

with $p = \sum p_n$ and with

$$\Gamma(p) = \frac{\Gamma_0}{2} \log \left(\frac{\cosh(p + p_{\max})}{\cosh(p - p_{\max})} \right). \quad (11)$$

The equations above can be put into the form of a set of ordinary differential equations and solved numerically using an o.d.e. solver. From the calculated pressure in the cavity and the volume velocity of the membrane, the pressure radiated by the clarinet can be calculated afterwards (see Appendix B for more details).

5.3 Results

Some results of numerical simulations are presented in Figure 8 with green lines and compared with experimental data (blue lines). Both simulations and experiments are obtained for a gain $\Gamma_0 = 1.5$ and for $L_{\text{cav}} = 3$ cm. As shown in Figure 8a, the fundamental frequency of self-oscillations obtained from numerical simulations are very close to those of the linear model (see Fig. 4) and as a result, again, the agreement with experiments is not perfect for the higher pitches. Both the experimental and simulated time signals for the pressure sensed in the cavity $u(t)/S_\mu$ and the one emitted away from the duct $p_{\text{rad}}(t)$ are shown in Figure 8b. Apart from the above mentioned disagreement on the fundamental frequency of self-oscillations, the results of numerical simulations show that both the amplitude and the shape of the signals are very well reproduced by numerical simulations, for all fingerings.

Additional simulations were performed to investigate the impact of the feedback gain Γ_0 . The results are shown in Figure 9 and should be compared to the experimental

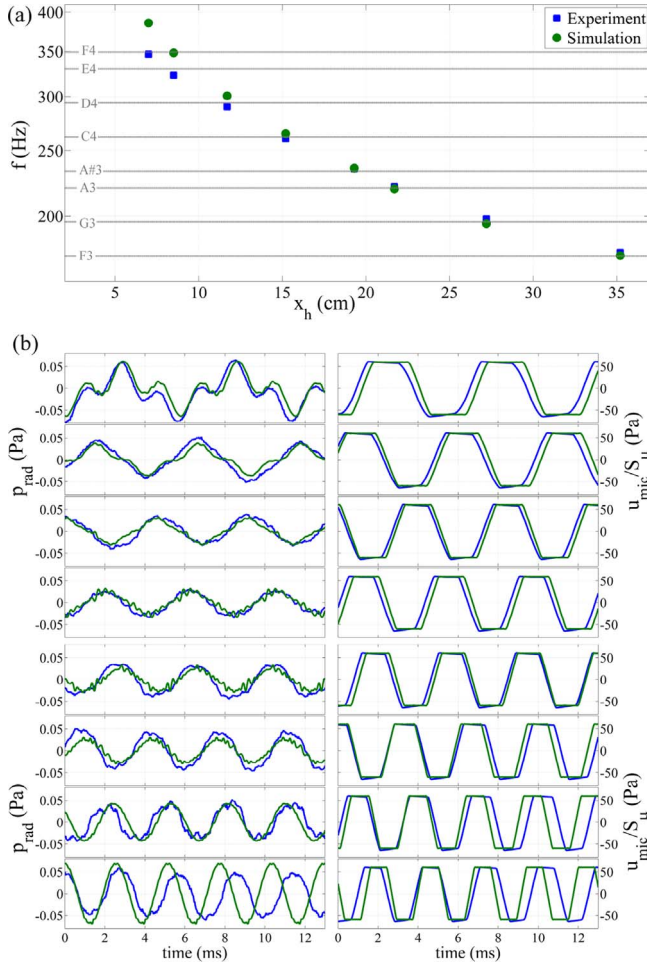


Figure 8. (a) Measured fundamental frequency (blue markers) of self-oscillations as functions of the applied fingering, for a feedback gain $\Gamma_0 \approx 1.5$, and for $L_{\text{cav}} = 3$ cm. The calculated fundamental frequencies of self-oscillations are also reported with green markers. (b) The corresponding experimental (blue lines) and simulated (green lines) acoustic pressure sensed by the feedback microphone (u/S_μ) or the one radiated 20 cm away from the open end of the duct p_{rad} , are also plotted for different fingerings, namely from all side holes closed on top, to all side holes open on the bottom.

results of Figure 5. Unlike experimental observations, the numerical simulations do not predict any significant change of the fundamental frequency as the gain Γ_0 increases. However, the increase in the amplitude of the third component observed in experiments as the gain increases is well reproduced by the model, as shown by the time signals for the steady-state radiated pressure $p_{\text{rad}}(t)$.

As a whole, the nonlinear model described in this paper captures the most important features of the dynamics observed in experiments. The main weakness of the model probably owes to the description of the nonlinear response of the microphone, where a black-box approach based on the measurement of a describing function is used here to characterize the amplitude-dependent response of the microphone. As a result, the reason for the observed discrepancy in the predicted and measured frequencies of

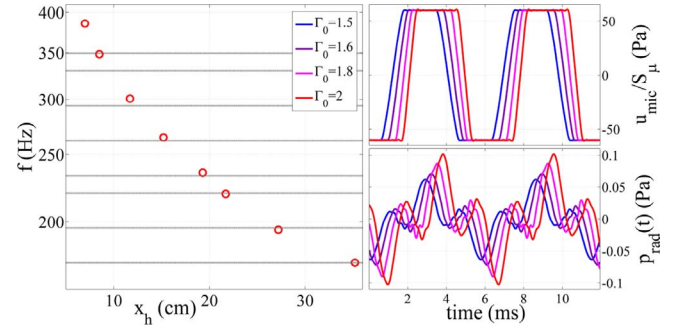


Figure 9. Calculated fundamental frequencies of the sound emitted as functions of the applied fingering, for different values of the feedback gain Γ_0 , and for $L_{\text{cav}} = 3$ cm. The acoustic pressure $p(t)$ in the cavity, and that emitted 20 cm away from the open end of the duct, $p_{\text{rad}}(t)$, are also plotted for the case of all side holes being kept closed.

self-oscillations is unclear and would appeal for further investigations.

6 Conclusion

In this paper, an experimental and theoretical study of a loudspeaker-driven clarinet was proposed. A linear model was derived, which can be used as first design tool for the instrument. A nonlinear model was also developed to predict the amplitude and frequency content of steady-state self-oscillations. It captures the essential dynamical features observed in experiments, but it fails in predicting with sufficient accuracy the frequency of self-oscillations or the increase of this frequency with an increasing gain. In practice, a wind instrument based on such an electroacoustic driving can be easily reproduced, and its tuning can be performed after some trial and error procedure to adjust the position of side holes. The authors doubt that the device in itself, or at least in its present form, could be used as a real musical instrument, but it is a nice demonstrator. It is also a good topic of study for a student in acoustics, as it involves to develop basic skills in signal processing and measurements, electroacoustics, transmission line theory, nonlinear dynamics, and numerical methods for finding roots or solving o.d.e.

The present study could be pursued in various ways. From a practical point of view, it might be interesting to find a simple way of disabling self-oscillations when necessary (without altering the gain Γ_0), for example by opening or closing an extra hole in the cavity, and it would also be interesting to try adding a register hole. Another perspective would be to try to connect the source (i.e. a loudspeaker, and a feedback loop in a cavity) to the body of a real musical instrument and to see if, by adjusting their coupling, the whole system would behave like a good-sounding clarinet. From a more theoretical or academic point of view, several other studies are possible, in particular to clarify the nonlinear behaviour discussed above. Another line to explore would be to study some synchronization phenomena by forcing this electroacoustic clarinet with an external sound

source, or by coupling two clarinets. Any of these complementary studies could possibly be carried on by a group of students, with the only goal of learning with fun.

Conflicts of interest

The authors declare no conflict of interest.

Data availability statement

The data are available from the corresponding author on request.

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Appendix A

Calculation of Z_{load}

The description of sound propagation through a network of ducts is a very common problem in acoustics. In the low-frequency range where the typical wavelength is much larger than the duct radius, it is quite usual to make use of a transmission line theory where each sub-element of the network is described as a two-port [14], which itself can be represented either with a matrix (e.g., a transfer matrix or a scattering matrix) or with an equivalent circuit. Here, we make use of the latter representation to describe the sound propagation through the acoustic load which consists here of the cavity connected to the duct drilled with a single side-hole, as shown in Figure A1. The extension to a duct drilled with several side-holes leads to a more complicated circuit, but it is straightforward.

The acoustic propagation through the cavity (length L_{cav} , radius r_{cav}) as well as the two ducts (length $L_{1,2}$, radius r_d) on both sides of the side holes can be described in terms of a T-shaped equivalent circuit as shown in Figure A1, where the impedances write as

$$Z_1 = Z_2 = iZ_c \tan\left(\frac{kL}{2}\right), \quad (\text{A1})$$

$$Z_T = \frac{Z_c}{i \sin(kL)}. \quad (\text{A2})$$

Those impedances account for the viscothermal losses through the expressions of the wave number k and the characteristic impedance Z_c . In the frame of a wide duct approximation, i.e. $\delta_v \ll r$, one has [15–17]

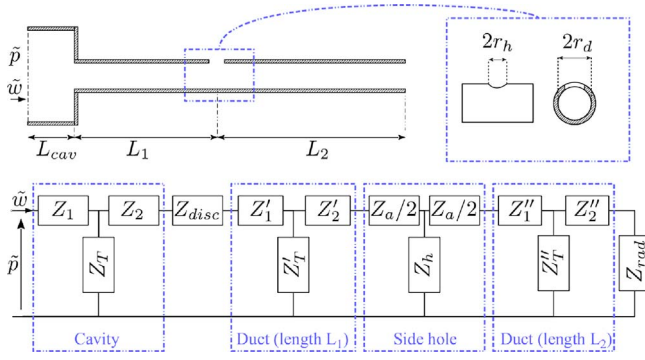


Figure A1. Electric analog of the acoustic load.

$$k = \frac{\omega}{c_0} \left[1 + (1 - i) \frac{\delta_v}{2r} \left(1 + \frac{\gamma - 1}{\sqrt{\text{Pr}}} \right) \right], \quad (\text{A3})$$

$$Z_c = \frac{\rho_0 c_0}{\pi r^2} \left[1 - (1 - i) \frac{\delta_v}{2r} \left(1 + \frac{\gamma - 1}{\sqrt{\text{Pr}}} \right) \right], \quad (\text{A4})$$

where r either refers to r_{cav} or r_d , where $\delta_v = \sqrt{2\nu/\omega}$ is the acoustic viscous boundary layer thickness, and where ν , γ and Pr stand for the kinematic viscosity, the Prandtl number and the specific heat ratio of the fluid, respectively.

As shown in Figure A1, the side hole can also be represented as a T-shaped electric analog, where the series impedance $Z_a = i\omega M_a$ represents some inertia effects described by means of an acoustic mass M_a (or equivalently by some length correction t_a) given by $M_a = \rho_0 t_a / (\pi r_d^2)$ with [18]

$$t_a = -0.28 r_h \left(\frac{r_h}{r_d} \right)^2 \quad (\text{A5})$$

where r_h stands for the radius of the side hole. The impedance of the hole, Z_h , is composed of a radiation resistance [16]

$$\Re(Z_h) = \frac{1}{4} \frac{\rho_0 c_0}{\pi r_h^2} \left(\frac{\omega r_h}{c_0} \right)^2, \quad (\text{A6})$$

and of an inductance [18–20]

$$i\Im(Z_h) = i\omega \frac{\rho_0 (t_i + t_d + t_r)}{\pi r_h^2}, \quad (\text{A7})$$

where t_d is the thickness of the duct, where the inner length t_i correction is given by [18]:

$$t_i = \left[0.82 - 1.4 \left(\frac{r_h}{r_d} \right)^2 + 0.75 \left(\frac{r_h}{r_d} \right)^{2.7} \right] \times r_h$$

and where the radiation length correction $t_r \approx 0.6133 r_h$ [21].

The discontinuity caused by the abrupt expansion at the connection between the cavity and the duct can also be taken into account by means of an inductance

$$Z_{\text{disc}} = i\omega \frac{\rho_0 \delta \ell}{\pi r_d^2}, \quad (\text{A8})$$

where the length correction $\delta \ell$ is given by [22]

$$\delta \ell = 0.82 r_d \left[1 - 1.35 \frac{r_d}{r_{\text{cav}}} + 0.35 \left(\frac{r_d}{r_{\text{cav}}} \right)^3 \right]. \quad (\text{A9})$$

Finally, the radiation at the open end of the duct is described through the impedance $Z_{\text{rad}} = R_{\text{rad}} + i\omega M_{\text{rad}}$ with [23]

$$R_{\text{rad}} = \frac{1}{4} \frac{\rho_0 c_0}{\pi r_d^2} \left(\frac{\omega r_d}{c_0} \right)^2$$

$$M_{\text{rad}} = \frac{\rho_0}{\pi r_d^2} [0.8216 - 0.2083\epsilon + 0.057\epsilon(1 - \epsilon^5)] r_d$$

where $\epsilon = r_d / (r_d + t_d)$ is the ratio of the internal radius to the external radius of the duct.

As illustrated in the electroacoustic analog of Figure A1, all the formula listed above can be used to calculate the load impedance Z_{load} , which is defined as the ratio of the pressure to the volume velocity at the input of the load. Also, the accuracy of this description can be checked experimentally. This was done here by the measurement of the impedance of the duct (without the cavity) with one or several side holes. The measurements were made using an acoustic impedance sensor [24, 25] and the results are shown for different fingerings in Figure B1 with solid red lines, as well as the ones of the model (dashed black lines). The results show that when all side holes are closed, the theoretical results almost coincide with the experiments, see Figure B1a. When one or several holes are open, see Figures B1b–B1d, the theoretical results are in very good agreement with experiments in the low frequency range (below 1 kHz), while the model seems to underestimate the losses for frequencies higher than 1 kHz. This could be explained by the fact that the model does not account for the external interactions between toneholes [26]. Considering that the fundamental frequency of self-oscillations does not exceed 500 Hz, the two-port model seems adequate for describing the acoustic load and the way it depends on the fingering.

Appendix B

Calculation details for the nonlinear model

This appendix describes how to get a set of o.d.e. from the equations governing the dynamics of the self-oscillator, namely equations (10) and (11), and how to evaluate the radiated pressure p_{rad} from the calculated pressure p in the cavity and the volume velocity w of the loudspeaker.

The re-arrangement of equations (10) and (11) in the form of a set of o.d.e. can be made by introducing the new functions f , g , q and r such that $f = \dot{w}$, $g = \dot{f}$, $q = \dot{p}$ and $r = \dot{q}$, where we remind that the pressure oscillations

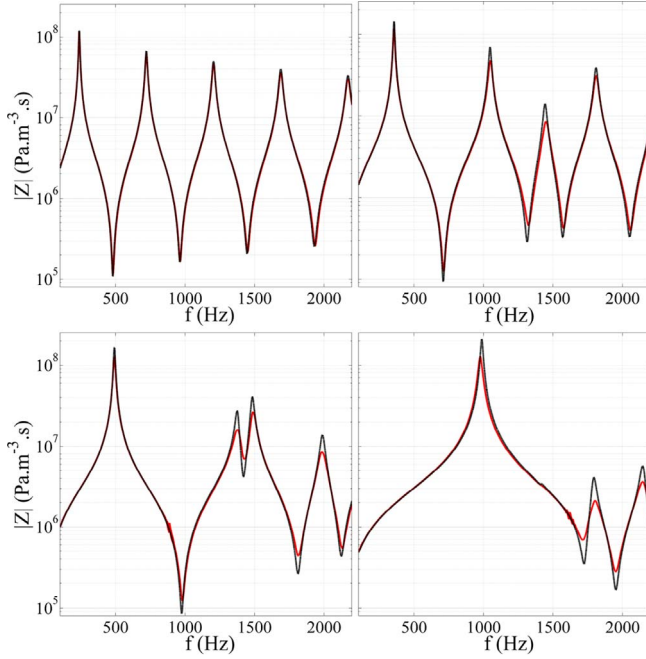


Figure B1. Comparison between the measured (solid red line) and the calculated (black dashed) input acoustic impedance of the duct (without the cavity) for different fingerings, defined as: (a) ●●●●●●, (b) ●●●●○, (c) ●●○○○○, (d) ○○○○○○ where a filled (resp. open) circle stands for a closed (resp. open) side hole. The hole positions are those given in Table 2.

can be decomposed (after applying a peak-picking algorithm) as the linear combination of N terms, namely $p = \sum p_n$. After some algebra, the following set of o.d.e. is obtained:

$$\begin{aligned}\dot{w} &= f, \\ \dot{f} &= g, \\ \dot{g} &= \mathcal{F}(w, f, g, p, q, r), \\ \dot{p}_1 &= q_1, \\ \dot{q}_1 &= r_1, \\ &\dots \\ \dot{p}_N &= q_N, \\ \dot{q}_N &= r_N,\end{aligned}$$

where $q = \sum q_n$, $r = \sum r_n$ and

$$r_n = A_n \dot{w} - 2\zeta_n \omega_n \dot{p}_n - \omega_n^2 p_n.$$

The nonlinearity is described by the function $\mathcal{F}$ defined as

$$\begin{aligned}-M_{\text{ma}} \mathcal{F} &= (\omega_0 M_{\text{ma}} + R_{\text{ma}})g + (\omega_0 R_{\text{ma}} + K_{\text{ma}})f \\ &+ \omega_0 K_{\text{ma}} w + \omega_0 q - \frac{d^2 \Gamma}{dt^2},\end{aligned}$$

where

$$\frac{d^2 \Gamma}{dt^2} = \frac{\Gamma_0 \Phi(p)}{2} \{r - q^2 [2 \tanh(p - p_{\text{max}}) + \Phi(p)]\},$$

and

$$\Phi(p) = \frac{\sinh(2p_{\text{max}})}{\cosh(p - p_{\text{max}}) \cosh(p + p_{\text{max}})}.$$

A numerical solution can be obtained using a o.d.e. solver, and gives access to the steady-state volume velocity of the membrane $w(t)$, to the real pressure in the cavity $p(t)$ and the one sensed by the feedback microphone, given by equation (6). For the need of comparison with experimental data, it is also worth calculating the pressure $p_{\text{rad}}(t)$ radiated at some distance d from the open side of the duct. This can be done from the knowledge of both $p(t)$ and $w(t)$, if one assumes that only the plane mode is propagating in the cavity and the duct, and that the propagation is linear. A Fourier series expansion is first applied to $p(t)$ and $w(t)$. Next, using a T-matrix matrix approach as described in Appendix A, the fundamental and higher harmonics components of pressure and volume velocity along the duct can be calculated. In particular, this gives access to the modulus and phase of each harmonic component of the volume velocity emitted by every single open hole. As a result, by making use of the superposition principle and by assuming that each open hole acts as a monopole source with a known volume velocity, the modulus and phase of each harmonic component of the pressure radiated at a given point in space can be calculated.

Appendix C

Results with a non-saturating feedback microphone

A simple way to support the fact that the feedback microphone is the leading source of sound saturation is to perform experiments with another microphone able to support much higher acoustic levels. In the following, the initial feedback microphone (ROGA MI19) was replaced by a 1/4" microphone GRAS 46BE supplied by a CCP power module GRAS 12AX. This microphone has a lower sensitivity ($\approx 4\text{mV/Pa}$) but it can support much higher sound pressure levels ($\approx 160\text{ dB SPL}$). Experiments were performed with this new microphone using the same protocol as the one described in Section 4.1, and the results obtained are presented in Figure C1, in addition to the ones obtained previously with a microphone ROGA MI19.

For both sets of measurements, the total gain of the feedback loop, Γ_0 , is set to its lowest value giving rise to self-oscillations (whatever the applied fingering). This threshold value of Γ_0 is about 1.5 when a ROGA MI19 is used as the feedback microphone, while $\Gamma_0 \approx 1.35$ when a GRAS 46BE is used. The reason why the threshold value of Γ_0 leading to self-oscillation is lower with a microphone GRAS than with a microphone ROGA is unclear, but it could be due to the difference in the low cut-off frequency of the two sensors. As observed in the upper part of Figure C1, it appears that the pitch of the sound emitted by the clarinet for various fingerings depends on the microphone used in the feedback loop. More precisely, with the same applied fingering, the fundamental frequency of self-oscillations is lower with a GRAS than with a ROGA.

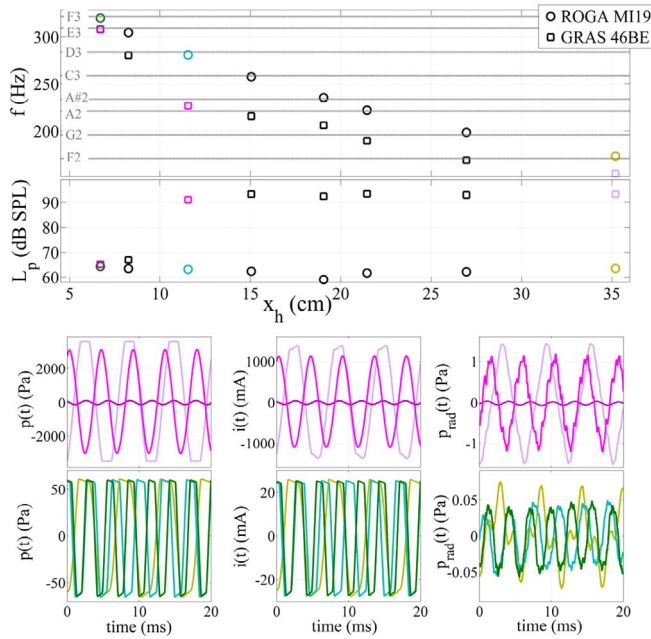


Figure C1. Measured fundamental frequency and sound pressure level L_p of the sound emitted 20 cm away from the open end of the duct as functions of the applied fingering, for $L_{cav} = 3$ cm and for both cases of a feedback loop that uses either a microphone ROGA MI19 ($L_{p_{max}} \approx 125$ dB SPL) or a microphone GRAS 46BE ($L_{p_{max}} \approx 160$ dB SPL). For all fingerings the overall feedback gain is constant, with $\Gamma_0 \approx 1.5$ when a ROGA MI19 is used as the feedback microphone, and $\Gamma_0 \approx 1.35$ when a GRAS 46BE is used. The acoustic pressure $p(t)$ in the cavity, the pressure $p_{rad}(t)$ emitted 20 cm away from the open end of the duct, and the current $i(t)$ supplied to the loudspeaker are also plotted for three different fingerings.

Again, a plausible explanation would come from some differences in the low-cut off frequency of the two sensors. The sound pressure level emitted 20 cm away from the open

end of the duct is also shown in Figure C1 for different fingerings, and the results show that the sound emitted is much louder (by ≈ 30 dB) with a microphone GRAS than with microphone ROGA (except for the two fingerings with the highest pitch). Considering that the feedback gain is almost the same for both cases, this is a clear confirmation that the distortion by the microphone ROGA is the main source of sound saturation, and therefore that a way to increase the level of the sound pressure emitted by the system is to change the feedback microphone. In this work, we did not make additional measurements with the microphone GRAS, mostly because there was a risk of destroying the loudspeaker. When a microphone GRAS is used for the feedback, the current supplied to the loudspeaker can indeed exceed 1 A, and the coil might be destroyed by the local heating caused by the Joule effect.

The lower part of Figure C1 shows the time signal of the pressure in the cavity, the current supplied to the loudspeaker, and the pressure radiated 20 cm away from the open end of the duct. Those temporal signals are shown for three different fingerings and for both cases of using a ROGA MI19 or a GRAS 46BE as the feedback microphone. The results show that there is no visible distortion of the acoustic pressure in the cavity when a GRAS 46BE is employed, except for the case of all side holes being closed (and in this latter case the visible saturation is rather due to the maximum input voltage of ± 5 V supported by the data acquisition card). Therefore, when a “non-saturating” microphone is used in the feedback loop, the nonlinear model presented in Section 5 should be re-visited to include other sources of sound saturation (the loudspeaker nonlinearities and the generation of vorticity close to the side holes and the open end should be considered).