



Higher-order modal parameter estimation and verification of cross-laminated timber plates for structural-acoustic analyses

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Abstract – Cross-Laminated Timber (CLT) is a wood composite that is popular due to its favourable stiffness-to-mass-density ratio and environmental benefits, among other positive attributes. This paper presents the estimation of higher-order modal parameters and their use in the verification and validation of an Equivalent Single-Layer (ESL) CLT model. The eigenvectors, eigenfrequencies, and modal damping ratios of 20 out-of-plane vibration modes were experimentally determined. The experimentally determined eigenfrequencies and eigenmodes were correlated and compared to those derived from a numerical model. The modal damping ratios were compared to damping loss factors derived from the power injection method. A broadband frequency view is also considered, with the comparison of experimental and numerical forced response models. As part of the verification and validation process, a framework of key indices and respective criteria is suggested and discussed in this contribution. Over the course of the paper, an ESL CLT model which derives its properties from a layerwise basis is validated in detail against experimental measurements. The results are intended to be of relevance to both structural and acoustic domains.

Keywords: Cross-laminated timber (CLT), Modal analysis, Vibro-acoustics, Homogenisation, Vibration serviceability

1 Introduction

Cross-Laminated Timber (CLT) is a renewable and environmentally friendly engineered wood product. CLT has already shown potential as an alternative stand-alone structural element to established construction materials derived from mineral resources, such as concrete and steel. This is due to CLT's relatively high stiffness-to-mass-density ratio as a building element. However, while this property correlates positively with static structural requirements, the same property correlates negatively with noise and vibration requirements.

Further challenges in the application of CLT as a structural element are the orthotropic nature of wood, a constituent material, and the overall inhomogeneous nature of CLT. These characteristics lead to more complex mechanical behaviour, posing additional challenges over standard isotropic and homogeneous materials, such as steel. However, this general anisotropic and inhomogeneous nature of CLT also provides additional design degrees of freedom, which can be utilised to overcome competing design requirements.

In order to exploit the more complex characteristics of CLT in design and realise the potential of the additional

design degrees of freedom available with such a structural composite, CLT must be thoroughly characterised and well understood. Field and laboratory testing, along with the development of analytical methods and numerical simulation models, go hand-in-hand to help achieve this and realise the full design potential of CLT or even similar products.

To date, the Equivalent Single-Layer (ESL) elastic properties of individual CLT panels have been characterised in references [1, 2] based on experimental modal analysis and updating of an analytical plate model. The values of the dynamically determined elastic stiffness coefficients were found to be approximately 5% stiffer than those derived by quasi-static bending testing. A result that is suggested to be due to the higher strain rate of the dynamic tests inducing a stiffening effect on the material [3]. This result has also been found in the dynamic characterisation of wood specimens [4, 5]. However, reference [6] elaborates that for quasi-static tests, creep effects are present as well as the local crushing of the wood fibres, which indicates that the stiffness moduli determined from dynamic testing, in the linear range, are more accurate than those determined from quasi-static testing [7]. Since modal analyses are generally conducted in the linear range, it may be assumed that the difference in quasi-static vs dynamically determined moduli is rather due to the effects of local crushing

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and creep present in quasi-static testing as opposed to the effect of the stiffening of the moduli due to the increased strain rate in modal testing [8, 9]. This postulation would, however, require further investigation for CLT.

Static and dynamically derived properties of CLT panels were further investigated in [10]. It was found that quasi-static bending tests on 300 mm wide CLT strips predicted the overall bending stiffness of a large CLT plate more reliably than narrower 100 mm wide strips due to the effects of local defects and non-homogeneities present in the material, such as knots, pitch pockets, cracks or delaminations of the timber. However, when the averages of 5 or 6 100 mm wide strips were considered, the results were close to those derived by the non-destructive modal analysis method of [1, 2], with a mean difference of 8% for all tested panels [10]. Reference [11] has determined the elastic properties of CLT by using both an eigenfrequency-based model updating method similar to that presented in references [1, 2, 10], acoustic testing, and quasi-static bending tests conducted on 300 mm wide CLT strips. It was found that for accurate determination of the elastic properties using an eigenfrequency-based model updating method required an adequate number of eigenfrequencies to be used in the cost function. A tangential idea of including higher-order eigenfrequencies to improve the accuracy of material property determination is supported in reference [12], where a better agreement was found when higher-order, rather than lower-order, eigenfrequencies are selected for the updating process.

While there has been an increasing number of studies characterising CLT while focusing on two modal parameters, namely the eigenmodes and the eigenfrequencies, there appears to be less literature on determining the modal damping properties of CLT. However, damping, along with the eigenmodes and eigenfrequencies of a structure, is a fundamental parameter for determining its dynamic response. Therefore, the damping properties of CLT elements must also be accurately characterised if the response of the structure is to be correctly determined. In the literature, reference [13] has analysed up to 7 modes, experimentally and numerically, for a CLT slab. For each of the 7 modes, the three modal parameters were determined, namely the eigenmodes and associated eigenfrequencies and modal damping ratios. The study found the experimentally determined eigenmodes to be complex, which could indicate that the slab has non-proportional damping mechanisms. However, this result appeared to be due to the slab being measured over the course of 2 days. It was suggested that the temperature variation over the course of the measurements resulted in a non-stationary system, thus violating an assumption of classical modal analysis, time invariance of the system, which resulted in significantly complex modes [14]. The effect of time invariance on the modal parameter estimation process was ameliorated by applying the central axis rotation method presented in reference [15]. This post-processing procedure reduced the complexity of the modes, achieving eigenmodes closer to those of real modes. Reference [16] has considered the damping ratios of 5-ply CLT plates for up to 6 modes under different boundary

conditions. It was found that two CLT plates joined together by screws and a half-lap joint and clamped along the shorter edges (CFCF) resulted in approximately a 20% increase in the damping ratio for the first fundamental bending mode, but an approximately 20% decrease for the third and higher modes with respect to a single CLT plate. When the CFCF case was compared to the analogous simply-supported case (SFSF), it was found that the CFCF case decreased the damping ratio of first-order modes by approximately 25% and generally increased the damping ratio for second- and third-order modes by 40%. Although, some second- and third-order modes also saw a decrease in the damping ratio. It was speculated that any reduction in damping due to the CFCF case was due to the minimisation of impact interaction with the supports. Reference [16] also studied damping ratios of up to 9 modes with 7-ply CLT floor systems; however, the eigenmodes above the 6th eigenmode are not visually identifiable and modal assurance checks have not been discussed in the paper.

So far, studies on determining the modal properties of CLT have focused on the eigenfrequencies and eigenmodes and have generally considered up to as many as 12 modes [1, 2, 10, 11]. However, the damping properties of the modes, which also constitute a fundamental parameter along with eigenmodes and eigenfrequencies in determining the response of a structure, have been less extensively studied in the literature. Studies determining the modal damping properties of CLT have been focused closer to the fundamental modes of the structure [13, 16]. This contribution extracts 20 eigenmodes along with their eigenfrequencies and damping ratios from experiments on two cross-laminated timber floor plates. The experimentally determined eigenmodes and associated eigenfrequencies are correlated and compared to those derived from a numerical model, while the experimentally determined modal damping ratios are compared to a one-third octave band average internal loss factor that is experimentally derived on an energy basis. This contribution additionally suggests and discusses key indices to consider and criteria to adhere to in order to ensure the quality of the fitted modal parameters and fidelity of theoretical models. This contribution also demonstrates that individual CLT plates exhibit real modes, and so are proportionally damped. Any complexity of the eigenmodes is due to artefacts of the curve fitting algorithm and so-called close modes. Further, this contribution considers mass normalised eigenmodes when correlating experimental and numerical results, which, to the Authors' knowledge, has not been utilised before for CLT. The mass normalisation ensures the orthogonality and so uniqueness of the eigenmodes, something that should ideally be incorporated into model updating routines to help ensure the uniqueness of a solution.

Another aspect of this contribution is the consideration of an ESL model whose properties have been derived from a layerwise model. Since CLT is a composite structure rather than an actual material that is macroscopically uniform, CLT can consist of a variety of constituent materials, geometries, number of plies, ply thickness, ply orientations and also stacking sequences, that is, how the aforementioned

factors are then combined to create a CLT element. If CLT is to be used to its full potential and advantage is to be taken of the aforementioned additional design degrees of freedom allowed by CLT's multilayer composite nature, a normalised basis of comparison is required. The utilisation of an ESL model allows for the determination of effective properties that can be normalised by the thickness of the considered element, thus providing a standardised basis of comparison. Such a basis of comparison can be used in the design process to compare different CLT elements and other construction materials. A model for deriving ESL properties from CLT on a layerwise basis has been detailed and validated in reference [17]. Reference [18] has provided further validation of this model. As an additional basis of comparison, this contribution presents and utilises the properties of CLT in terms of ESL coupled extension-bending stiffness matrices in addition to the derived ESL elastic moduli and Poisson's ratios that are presented and utilised in references [17, 18]. This contribution primarily extends on reference [18] with a focus on determining modal parameters and verification and validation of the numerical models in the modal domain. The complementary use of both modal and response models for the verification and validation of theoretical models for vibro-acoustic problems is discussed. However, this contribution is expected to not be just applicable to the field of vibro-acoustics, but also to the field of vibration serviceability. The contribution may also be relevant for seismic design cases where damage to non-structural elements from minor seismic events is of interest.

2 Background and theory

2.1 Calculation tools

2.1.1 Modal analysis

The modal model utilised in this contribution is a symmetric undamped model [19]

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{0}, \quad (1)$$

where $\mathbf{K}, \mathbf{M} \in \mathbb{R}^{n \times n}$ are the stiffness and mass matrices of the system, respectively. $\mathbf{u}, \ddot{\mathbf{u}}, \mathbf{0} \in \mathbb{C}^n$ are the displacement, acceleration, and zero vectors of the system, respectively. $n \in \mathbb{N}$ is the number of degrees of freedom of the model.

2.1.2 Harmonic analysis

The response model utilised in this contribution is a forced harmonic response model with hysteretic damping. The equation of motion is [19]

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}(1 + i\eta)\mathbf{u} = \mathbf{F}, \quad (2)$$

where $\mathbf{K}, \mathbf{M} \in \mathbb{R}^{n \times n}$ are the stiffness and mass matrices of the system, respectively. $\mathbf{u}, \ddot{\mathbf{u}}, \mathbf{F} \in \mathbb{C}^n$ are the displacement, acceleration, and external loading vectors of the system, respectively. The externally applied loads may be a combination of frequency-dependent harmonic and static loads. $n \in \mathbb{N}$ is the number of degrees of freedom

of the model, and $\eta \in \mathbb{R}$ is the structural damping loss factor of the system. i is the imaginary unit.

2.1.3 Homogenisation

The homogenisation theory applied in this contribution is based on an augmented classical lamination theory that includes first-order shear deformations with shear correction factors derived from the actual shear stress distribution of the considered laminate. The homogenised properties are given in stiffness or compliance matrix form [17],

$$\begin{bmatrix} \mathbf{N} \\ \mathbf{M} \\ \mathbf{V} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} & \mathbf{0} \\ \mathbf{B} & \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \tilde{\mathbf{H}} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon} \\ \boldsymbol{\kappa} \\ \boldsymbol{\gamma} \end{bmatrix}, \quad (3)$$

where $\mathbf{A}, \mathbf{B}, \mathbf{D} \in \mathbb{R}^{3 \times 3}$ are the extensional stiffness, bending stiffness, and bending-extensional coupling stiffness matrices, respectively. $\tilde{\mathbf{H}} \in \mathbb{R}^{2 \times 2}$ is the shear-corrected transverse shear stiffness matrix. $\mathbf{N}, \mathbf{M}, \boldsymbol{\varepsilon}, \boldsymbol{\kappa} \in \mathbb{R}^3$ are the in-plane force, bending moment, in-plane strain, and curvature vectors, respectively. $\mathbf{V}, \boldsymbol{\gamma} \in \mathbb{R}^2$ are the transverse shear force and transverse shear strain vectors, respectively. The uncoupled flexural and planar engineering constants can be derived from the stiffness matrices [17].

2.1.4 Power injection method

A means for determining an averaged loss factor on an energy basis of a vibrating structural system is the power injection method [20]. The power injection method approximates the loss factor $\eta \in \mathbb{R}$ of the system by the ratio of the dissipated energy $E_{\text{diss}} \in \mathbb{R}$ to the maximum strain energy $E_V \in \mathbb{R}$ of the system per radian under steady-state vibration [21],

$$\eta = \frac{E_{\text{diss}}}{2\pi E_V}. \quad (4)$$

Assuming the input force consists of a stationary signal and remains fixed at a location, the energy dissipated is equivalent to the energy input into the system [21, 22],

$$E_{\text{diss}} = E_{\text{in}}. \quad (5)$$

The energy input into the system can be derived at a given frequency from the power input into the system W_{in} (input force multiplied by velocity at the point of excitation, determined from the cross power spectrum) multiplied by the period of the given cycle $T \in \mathbb{R}$ [21],

$$E_{\text{in}} = W_{\text{in}} T. \quad (6)$$

The total strain energy can be approximated from the total kinetic energy of the system. An approximation that is valid at resonance or when considering a statistical average across resonances in a given frequency band. The total kinetic energy for a uniform plate can be calculated from the temporally and spatially averaged squared velocity response of the system presented in equation (11) [21],

$$E_V = E_T = \frac{1}{2} m \overline{|v|^2}. \quad (7)$$

Substituting equations (5)–(7) into equation (4), the loss factor is defined by terms that can be conveniently determined in situ:

$$\eta = \frac{2W_{in}}{\omega m \overline{|v|^2}}. \quad (8)$$

2.2 Comparison and correlation tools

2.2.1 Modal domain

Frequency response functions, while valuable for model comparison and correlation, are not necessarily unique [23]. However, the mass-weighted eigenmodes and associated eigenfrequencies are unique for the case of a system with proportional damping, as considered in this contribution [19, 23]. Therefore, this contribution considers correlation in the modal domain with shape-based (eigenvector) and scalar-based (eigenvalue) indices for the eigenmodes and eigenfrequencies, respectively.

Shape based index

The Modal Assurance Criterion (MAC) is a useful index in the field of vibro-acoustics to not only compare eigenmodes, but to ensure that the correct eigenfrequency-eigenmode pairs are identified. The MAC is a statistical concept defined as a single scalar constant relating the degree of consistency between eigenmodes [24]. The MAC is essentially a squared linear regression correlation coefficient and so is based upon the minimisation of the squared error between two vector spaces. A mass-weighted modal assurance criterion (WMAC) is considered in this contribution, calculated by [23]

$$WMAC(\psi_i, \psi_j) = \frac{|\psi_i^H \mathbf{M} \psi_j|^2}{|\psi_i^H \mathbf{M} \psi_i| |\psi_j^H \mathbf{M} \psi_j|}, \quad (9)$$

where $WMAC \in [0, 1]$. $\psi_i, \psi_j \in \mathbb{C}^n$ are the $n \in \mathbb{N}$ degree of freedom modal vectors $i, j \in \mathbb{N}$. The operator H is the Hermitian transpose and $\mathbf{M} \in \mathbb{R}^{n \times n}$ is the mass matrix associated with either of the modal vectors.

The mass matrix weighted WMAC, as considered here in equation (9), is additionally a pseudo-orthogonality check of the considered eigenmodes. The WMAC values range between zero and one, indicating that the eigenmodes are completely uncorrelated/orthogonal or correlated/parallel with the caveat that a sufficient number of degrees of freedom have been considered in the calculation of the WMAC. In practice, no precise criteria are prescribed for the values of the WMAC to indicate what constitutes a correlated eigenmode pair and an uncorrelated eigenmode pair. Typically, WMAC values of 0.9 or greater are considered to be well-correlated eigenmode pairs and values of 0.1 or less are considered to be uncorrelated eigenmode pairs [23, 24].

Scalar based index

The Percentage Error (PE) gives the percentage-wise relative difference of one value with respect to another. In this contribution, the PE is used to compare numerically determined eigenfrequencies with respect to experimentally determined eigenfrequencies. The general PE of the eigenfrequencies is,

$$PE(f_{a,i}, f_{e,j}) = \left(\frac{f_{a,i} - f_{e,j}}{f_{e,j}} \right) \times 100\%, \quad (10)$$

where $PE \in \mathbb{R}$. $f_{a,i}, f_{e,j} \in \mathbb{R}$ are the $i, j \in \mathbb{N}$ eigenfrequencies for the respective numerically and experimentally determined solution sets $a, e \in \mathbb{N}$.

The criterion applicable to the PE is strongly dependent on the field and application. A criterion of $PE < 10\%$ was set in this contribution. This criterion was chosen based on one-third octave bands, which are commonly used in building acoustic analyses [25]. The half-bandwidth of a given one-third octave band is 11.5% of its exact mid-band frequency value, so the chosen criterion provides some assurance that the numerical eigenfrequencies will fall within the same one-third octave band as the experimental eigenfrequencies.

2.2.2 Frequency domain

While the modal domain indices discussed in Section 2.2.1 have the advantage of comparing theoretically unique parameters of the system, they are difficult to apply when considering the broadband dynamic behaviour of a structure as the response can involve thousands or more eigenmodes and large modal overlap factors. As a result, for broadband comparisons, the model is compared in the frequency domain with an energy-based (response) index rather than dealing with the details of the individual eigenmodes and associated eigenfrequencies.

Energy based index

The temporal and spatial average of the square of the surface response velocity $\overline{|v|^2} \in \mathbb{R}$ is a useful index for broadband structural-acoustic response comparisons. $\overline{|v|^2}$ is defined as [21]

$$\overline{|v|^2} = \frac{1}{m_{tot}} \iint \rho_A(x, y) |v(x, y)|^2 dx dy, \quad (11)$$

where $m_{tot} \in \mathbb{R}$ is the total mass, $\rho_A(x, y) \in \mathbb{R}$ is the area density, and $v(x, y) \in \mathbb{C}$ is the complex velocity distribution of the vibrating system. $\overline{|v|^2}$ is the approximate total energy of the vibrating system divided by the total mass of the system at resonance, where bending waves are dominant.

In order to compare $\overline{|v|^2}$ across different studies, $\overline{|v|^2}$ is normalised by the square of the modulus of the respective point excitation force $F_i \in \mathbb{C}$ to derive the spatially averaged squared transfer mobility $\overline{|Y|^2}$,

$$\overline{|Y(\omega)|^2} = \frac{\overline{|v(\omega)|^2}}{\overline{|F_i(\omega)|^2}}. \quad (12)$$

2.2.3 Matrix condensation

To obtain the mass matrix for normalisation of the eigenmodes in Section 2.2.1, the numerically determined mass matrix is condensed to the measurement degrees of freedom of the experimental model by means of Guyan reduction. The vectors and matrices in equation (1) can be partitioned into kept degrees of freedom $\mathbf{u}_1$ and eliminated degrees of freedom $\mathbf{u}_2$ giving

$$\mathbf{u} = \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix}, \quad (13)$$

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix}, \quad (14)$$

and

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix}. \quad (15)$$

The reduced mass matrix is then obtained by applying the following transformation [19],

$$\mathbf{M}_{11} = \begin{bmatrix} \mathbf{I}_{11} \\ -\mathbf{K}_{22}^{-1}\mathbf{K}_{21} \end{bmatrix}^T \begin{bmatrix} \mathbf{M}_{11} & \mathbf{M}_{12} \\ \mathbf{M}_{21} & \mathbf{M}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{11} \\ -\mathbf{K}_{22}^{-1}\mathbf{K}_{21} \end{bmatrix}, \quad (16)$$

where $\mathbf{I} \in \mathbb{N}^{n_1 \times n_1}$ is the identity matrix. The result of equation (16) generally leads to a dense mass matrix. However, the mass matrices in modal analysis models are diagonal lumped-mass matrices [19]. As a result, the row sum mass-lumping method is applied to the reduced mass matrix in order to obtain a diagonal mass matrix consistent with the modal models [26]

$$\bar{M}_{ii} = \sum_{j=1}^n M_{ij} \quad \forall i = \{1, \dots, n\}. \quad (17)$$

3 Materials and methods

3.1 Test specimens

The test specimens considered were two 5-ply CLT plates with $[0/90/0/90/0]$ lay-up (or concisely, $[0/90/0]_s$) and plies of 40 mm thickness each. Both specimens had a $5.73 \text{ m} \times 0.100 \text{ m} \times 0.100 \text{ m}$ notch for a half-lap joint along one side. The geometrical properties of the plates are given in Table 1.

3.2 Experimental set-up and measurement

Both of the plates had similar experimental set-ups, supported on three air jacks while they were excited on the underside with an electrodynamic shaker and scanned from above with a Polytec PSV-400 laser Doppler vibrometer. A photo of the experimental set-up with CLT200b is presented in Figure 1. A photo of the experimental set-up for CLT200a can be found in reference [17]. CLT200a was excited with a Data Physics IV40 inertial shaker, while

CLT200b was excited with a Bruel and Kjaer LSD V201 modal shaker. At their respective excitation points, both plates had a PCB 288D01 impedance head attached, to which the respective shakers were attached.

For CLT200a, sets of coarse and fine measurements were conducted for three different excitation points each. The approximate excitation locations are depicted in Figure 2a, and the relative excitation point locations are given in Table 2. The excitation signal was a swept sine from 1 Hz to 5500 Hz and 16 Hz to 6600 Hz for the coarse and fine measurements, respectively. The measurement grid for the coarse measurements was a rectangular 6×18 grid of points, and the fine measurement was a rectangular 16×39 grid of points, corresponding to spatial resolutions of approximately 39 cm and 15 cm, respectively. The approximate fine measurement grid is depicted in Figure 2a. Time sampling parameters of the measured signals were chosen to obtain FFT data with frequency resolutions of 195 mHz and 62.5 mHz for the coarse and fine measurements, respectively.

For CLT200b, the measurement grid was a 7×18 grid of points, corresponding to a spatial resolution of approximately 34 cm. The approximate measurement grid is depicted in Figure 2b, along with the approximate excitation locations where the relative positions are given in Table 2. Time sampling parameters of the measured signals were chosen to obtain FFT data with a frequency resolution of 195 mHz.

3.3 Modal parameter estimation

The experimental modal parameters presented in this contribution were extracted using the least squares complex exponential method [19, 23, 27]. The complex components of the respective eigenmodes were converted to real modes via a slightly augmented version of the “simple method” described in reference [23]. The augmentation was that the modal vectors were rotated by the mean phase angle [28] in the polar plane to minimise their respective distances to the line through 0° and 180° .

A stabilisation diagram was utilised to determine the correct model order for fitting of the modal model and to eliminate the occurrence of spurious modes, for example, modes due to noise or mathematical modes due to overfitting of the modal model. The following criteria were specified for stable eigenfrequencies $f \in \mathbb{R}$ and modal damping ratios $\zeta \in \mathbb{R}$ [27];

$$\frac{|f_{i,N} - f_{i,N-1}|}{f_{i,N-1}} < 0.01 \quad (18)$$

and

$$\frac{|\zeta_{i,N} - \zeta_{i,N-1}|}{\zeta_{i,N-1}} < 0.05, \quad (19)$$

where $N \in \mathbb{N}$ is the order of the model and $i \in \mathbb{N}$ where $i \leq N$ is the eigenvector index.

The so-called coarse measurement datasets described in Section 3.2 were used for the modal parameter estimation

Table 1. Identification (ID) of the experimentally investigated plates by geometrical properties. Namely length a , width b , total thickness t_{tot} , lay-up, and layer thickness t_i .

Plate ID	a (m)	b (m)	t_{tot} (m)	Lay-up ($^\circ$)	t_i (mm)
CLT200a	5.73	2.38	0.200	$[0/90/0]_s$	40
CLT200b		2.28			



Figure 1. Photo of the experimental set-up for plate CLT200b with shaker located at the F_1 excitation point.

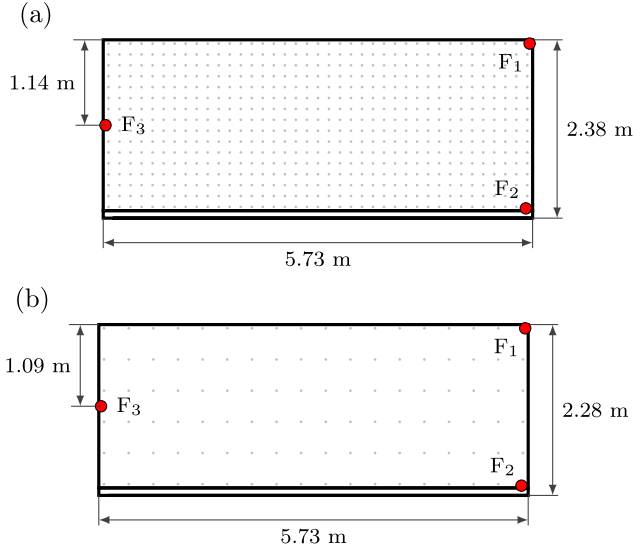


Figure 2. Schematic indicating planar dimensions, approximate measurement grid, and locations of the excitation positions for plates: (a) CLT200a and (b) CLT200b.

process of CLT200a due to the ease of processing with the smaller but adequate datasets.

3.4 Material property derivation

3.4.1 Mass density

Both plates were weighed, and the difference in their derived mass densities was less than 1%.

Table 2. Excitation point locations on the plane of the plates with respect to the nearest short edge Δx_b and long edge Δy_a .

Plate ID	Excitation point	Δx_b (cm)	Δy_a (cm)
CLT200a	F_1	4.00	5.00
	F_2	9.00	13.0
	F_3	3.00	114
CLT200b	F_1	4.00	5.00
	F_2	9.00	13.0
	F_3	3.00	109

3.4.2 Structural damping

The structural damping loss factor was derived on a one-third octave band basis from the experimental results using equation (8). The spatially averaged mean square velocity was obtained from the laser Doppler vibrometer measurements, and the power injected was determined from the cross-spectrum of the force and acceleration signals of the impedance head at the respective excitation points of the plates. The loss factor is only considered for bands with modes present.

The modal damping ratios of 20 modes were obtained with the modal parameter estimation procedure described in Section 3.3. The damping ratios are based on a viscous damping model, which represents the actual hysteretic damping of the structure as an equivalent viscous damping ratio. The modal damping ratio can be related to the loss factor by $\eta = 2\zeta_i$ [23]. However, this expression is only valid at resonance, and for small damping ratios due to the different forms of the material (hysteretic) and viscous damping hysteresis loops.

In general, the calculated damping factors capture not only internal mechanisms of damping but also external mechanisms, such as losses due to interaction with the air jacks and acoustic radiation. It is assumed that these external damping mechanisms are negligible, so the derived damping factors represent the internal losses of the system. However, the damping losses due to acoustic radiation can be non-negligible in the region of the critical frequency [29]: The frequency where the bending wavelength of the plates matches the airborne wavelength of the fluid. The critical frequencies of the plates were calculated to be $f_{c,x} = 70$ Hz for the x -direction and $f_{c,y} = 120$ Hz for the y -direction [21].

3.4.3 Engineering constants

The layerwise material properties used in this study were derived from a model updating process based on the experimentally determined eigenmodes and associated eigenfrequencies of CLT200a [17]. The layerwise and ESL properties are presented in Table 3.

For the ESL, the extensional stiffness $\mathbf{A}$, bending-extensional coupling stiffness $\mathbf{B}$, bending stiffness $\mathbf{D}$, and corrected transverse shear stiffness $\tilde{\mathbf{H}}$ matrices define the properties of the homogenised CLT plates. These stiffness matrices were derived from the layerwise properties presented in Table 3 using the homogenisation method in reference [17],

Table 3. Layerwise and ESL properties of the CLT plates.

Property	Units	Model	
		Layerwise	ESL
Lay-up	°	$[0/90/\bar{0}]_s$	$[0]$
Layer thickness	mm	40	200
Total thickness	mm	... 200 ...	
Loss factor, η	1	... 0.0170 ...	
Density, ρ	$\text{kg} \cdot \text{m}^{-3}$	... 444 ...	
Elastic moduli			
E_x	GPa	11.8	9.56
E_y	GPa	0.999	3.25
E_z	GPa	... 0.550 ...	
Poisson's ratios			
ν_{xy}	1	0.0100	0.0308
ν_{yz}	1	... 0.300 ...	
ν_{xz}	1	0.0100	0.0308
Shear moduli ^a			
G_{xy}	GPa	... 0.748 ...	
G_{yz}	GPa	0.176	0.158
G_{xz}	GPa	0.538	0.229

^a The corrected shear moduli are presented for the ESL.

$$\mathbf{A} = \begin{bmatrix} 1.50 & 0.0200 & 0 \\ 0.0200 & 1.06 & 0 \\ 0 & 0 & 0.150 \end{bmatrix} \text{GPa} \cdot \text{m}$$

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{Pa} \cdot \text{m}^2$$

$$\mathbf{D} = \begin{bmatrix} 6.37 & 0.0667 & 0 \\ 0.0667 & 2.17 & 0 \\ 0 & 0 & 0.499 \end{bmatrix} \text{MPa} \cdot \text{m}^3$$

$$\tilde{\mathbf{H}} = \begin{bmatrix} 31.5 & 0 \\ 0 & 45.8 \end{bmatrix} \text{MPa} \cdot \text{m}.$$

The flexural ESL engineering constants in Table 3 can be derived from the presented stiffness matrices [17]. These engineering constants are representative so long as the coupled bending-extensional behaviour of the plates is considered negligible, which is the case in this contribution. If the coupled bending-extensional behaviour of plates is considered non-negligible, then the given stiffness matrices should be utilised.

3.5 Numerical analyses

3.5.1 Modal analysis

The undamped modal analyses were conducted with the Finite Element Method (FEM) using Ansys Mechanical APDL, release 2022 R2.

Plate CLT200a was modelled on $\Omega \in \mathbb{R}^2$, where Ω was the spatial domain in the Cartesian plane based on the dimensions presented in Table 1

$$\Omega = [0, 5.73] \times [0, 2.38], \quad (20)$$

with boundary $\partial\Omega$ set to have free boundary conditions:

$$\mathbf{u} = \mathbf{u} \quad \forall \mathbf{u} \text{ on } \partial\Omega. \quad (21)$$

The boundary conditions were set under the assumption that the air jacks did not influence the response of the structure. The area models were then discretised using a mapped quadrilateral mesh with a global size of 40 mm

$$\Omega = \bigcup_{e=1}^{n_e} \Omega_e, \quad (22)$$

where $n_e \in \mathbb{N}$ is the number of mesh elements. The element used for the study was the Ansys SHELL181 element, a 4-node structural shell element. A shell element was chosen as it produces the same results as solid-shell and solid elements for the frequency range considered in the modal analysis but is more computationally efficient [17]. The shell section and properties were defined using a preintegrated general shell section with the stiffness matrices given in Section 3.4.3. However, it should be noted that when specifying the preintegrated transverse shear stiffness matrix in Ansys, the order of the stiffness coefficients is swapped compared to the $\tilde{\mathbf{H}}$ matrix in Section 3.4.3. That is, the matrix was specified as $H_{11} = 45.8 \text{ MPa} \cdot \text{m}$, $H_{22} = 31.5 \text{ MPa} \cdot \text{m}$, and $H_{12} = H_{22} = 0$ in Ansys. The density of the shell section was input as an area density and the default Ansys key options were used for the element. The half-lap joint notch seen in Figures 1 and 2 was not modelled. The plate was modelled as having a constant thickness.

For post-processing, the bending and torsional modes were selected and mapped to the coarse experimental measurement grid of plate CLT200a, see Sections 3.2 and 3.3.

Plate CLT200b was modelled and analysed in the same manner as plate CLT200a, with the exception that the spatial domain of plate CLT200b was defined as

$$\Omega = [0, 5.73] \times [0, 2.28], \quad (23)$$

and the results were mapped to the respective measurement grid of plate CLT200b.

3.5.2 Forced response analysis

The forced response analyses were conducted as harmonic analyses with hysteretic damping using Ansys Mechanical APDL, release 2022 R2.

Plate CLT200a was modelled on $\Omega \in \mathbb{R}^3$, where Ω was the spatial domain in Euclidean space with a Cartesian coordinate system based on the dimensions presented in Table 1

$$\Omega = [0, 5.73] \times [0, 2.38] \times [0, 0.20], \quad (24)$$

with boundary $\partial\Omega$ set to have free boundary conditions:

$$\mathbf{u} = \mathbf{u} \quad \forall \mathbf{u} \text{ on } \partial\Omega. \quad (25)$$

Similar to the modal analyses, the boundary conditions were set under the assumption that the air jacks did not influence the response of the structure. The volume models

were then discretised using a mapped hexahedral mesh with a global size of 20 mm

$$\Omega = \bigcup_{e=1}^{n_e} \Omega_e, \quad (26)$$

where $n_e \in \mathbb{N}$ is the number of mesh elements. The element used for the study was the Ansys SOLSH190 element, an 8-node structural solid-shell element. The SOLSH190 element was selected for its ability to capture through-thickness resonances of the plates, an effect that occurs at high frequencies in CLT panels [17]. An orthotropic material definition with the ESL material properties from Table 3 was used. The default Ansys key options were used for the element except for setting the shear correction factors to 1 since the ESL values presented in Table 3 already have the respective shear correction factors applied. Ansys, by default, includes a 5/6 shear-correction factor. The half-lap joint notch seen in Figures 1 and 2 was not modelled, with the plate having a constant thickness.

Analyses were then conducted for each of the three harmonic point excitation forces

$$F = \{F_i \in \mathbb{R} \mid F_i = 1\}_{i \in \{1,2,3\}}, \quad (27)$$

acting on the node nearest to the respective excitation point location, see Sections 3.2. The value of 1 N was chosen in order to obtain the transfer mobilities directly from the analysis. The swept sine excitation from the inertial shaker was modelled with logarithmic frequency intervals $f: \mathbb{R}^2 \times \mathbb{N} \rightarrow \mathbb{R}$ defined by

$$f = 10^{(\log_{10}(f_b) + (i-1)f_{inc})} \quad \forall i \in \{1, \dots, n\} : n \in \mathbb{N}, \quad (28)$$

where

$$f_{inc} = \frac{\log_{10}(f_e) - \log_{10}(f_b)}{n - 1}, \quad (29)$$

with $f_b = 6$ Hz, $f_e = 5657$ Hz, and $n = 1100$ substeps. The harmonic response was calculated using the full solution method.

For post-processing, the velocity responses were mapped to the experimental measurement grids of the respective plates before calculating the spatially averaged squared transfer mobilities. The fine-resolution measurement datasets in Sections 3.2 were used as the reference datasets for the forced response analysis for plate CLT200a.

Plate CLT200b was modelled and analysed in the same manner as plate CLT200a, with the exception that the spatial domain of plate CLT200b was defined as

$$\Omega = [0, 5.73] \times [0, 2.28] \times [0, 0.20], \quad (31)$$

and the results were mapped to the respective measurement grid of plate CLT200b.

4 Results and discussion

4.1 Modal analyses

Table 4 presents the eigenvalue results of the Experimental Modal Analysis (EMA) and Numerical Modal Analysis (NMA) for plate CLT200a. Figure 3 presents the

Table 4. Eigenvalues for CLT200a. Namely the experimental $f_{e,i}$ and numerical $f_{a,i}$ eigenfrequencies of the i th mode and their percentage error PE_i , plus the experimental modal damping factor $\zeta_{e,i}$ of the i th mode.

i	$f_{e,i}$ (Hz)	$f_{a,i}$ (Hz)	PE_i (%)	$\zeta_{e,i}$ (10^{-2})
1	19.0	19.0	0.00	0.75
2	28.3	28.3	0.00	0.36
3	46.3	46.6	0.65	0.59
4	74.3	73.3	-1.35	1.65
5	88.6	89.4	0.90	0.58
6	91.3	91.6	0.33	0.68
7	97.9	97.6	-0.31	0.58
8	116	116	0.00	0.60
9	130	132	1.54	0.95
10	143	145	1.40	1.00
11	149	149	0.00	0.69
12	194	196	1.03	1.10
13	196	198	1.02	1.03
14	207	209	0.97	1.30
15	211	217	2.84	0.95
16	216	221	2.31	0.81
17	229	234	2.18	0.86
18	250	252	0.80	1.00
19	254	257	1.18	1.10
20	266	267	0.38	0.96

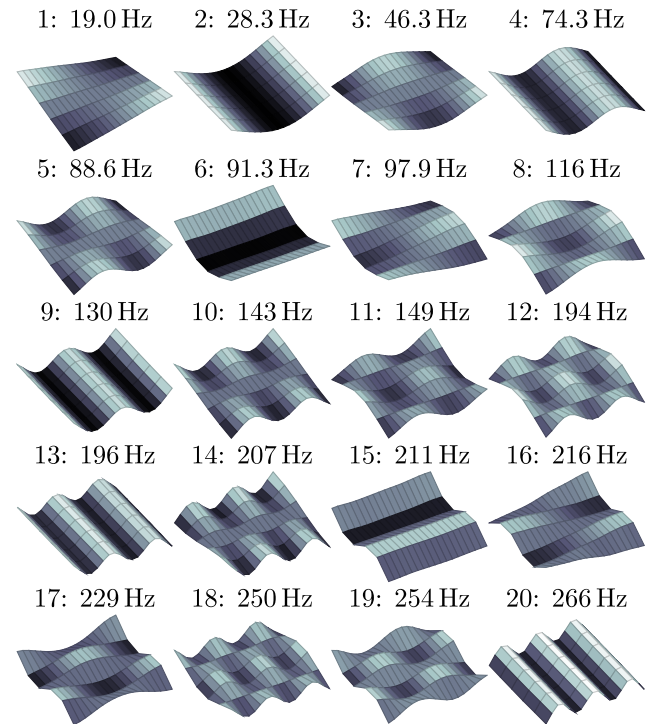


Figure 3. The first twenty identified eigenmodes with their experimental eigenfrequencies for CLT200a.

EMA-identified eigenmodes. It should be noted that for modes 13 and 15, the modal damping ratio convergence criterion had to be relaxed to 0.11 and 0.06, respectively, for equation (19) to be satisfied. This was likely due to the closeness of the modes. The eigenfrequency criterion in equation (18) was satisfied for all modes.

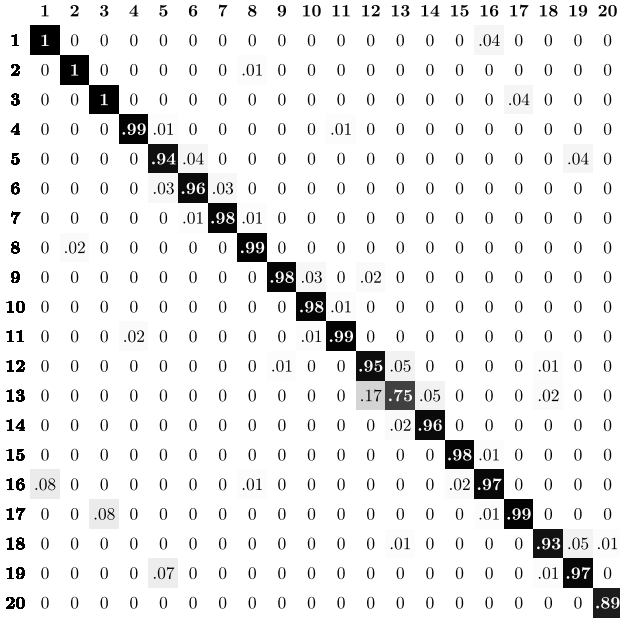


Figure 4. WMAC matrix of experimental (vertical) against numerical (horizontal) eigenmodes for CLT200a. The colourmap is a linear scale from white ($WMAC=0$) to black ($WMAC=1$).

Figure 4 presents the WMAC of experimentally fitted eigenmodes against the numerically determined real eigenmodes for plate CLT200a. The highest values of the WMAC indicate corresponding eigenmode pairs. As the highest correlations lie along the diagonal, the order of the eigenmodes of the EMA and NMA agree. Based on the WMAC criteria discussed in Section 2.2.1, 18 of the 20 modes are well correlated with $WMAC \geq 0.9$. The two modes that do not meet this criterion of being well correlated are the 13th and the 20th modes, which have values of 0.75 and 0.89, respectively. Referring to Table 4, Mode 13 is close in eigenfrequency to mode 12 with a difference of 1%. When the eigenfrequencies are separated by an amount that is less than the modal damping ratios, the modes are considered “close modes” [23]. When close modes occur, proportionally damped systems can also exhibit significantly complex eigenmodes. This is the case for modes 12 and 13, with a separation of 1.0% and $\zeta_{e,12} = 1.1\%$. An almost real mode is found for 12, but mode 13 is masked by the closeness of mode 12, resulting in a significantly complex mode. The complex-to-real conversion method used in this contribution (Sect. 3.3) is only robust for phase angle differences of $<10^\circ$. Hence, when converting significantly complex experimentally fitted modes, such as mode 13, to real modes for correlation with actual, real numerically determined modes, these modes are not expected to meet the criterion of a well-correlated mode [23]. This is also applicable to a lesser degree to mode 20 and mode 21 (not presented in this contribution), where the eigenfrequencies are separated by approximately 3%, with the contribution of mode 21 partially masking mode 20. This partial closeness of the modes also accounts for differences between a unity WMAC value and those seen for the well-correlated modes in Figure 4.

Considering the off-diagonal terms in Figure 4, the modes are considered to not be correlated with each other based on the criterion discussed in Section 2.2.1 with $WMAC \leq 0.1$. One exception is $WMAC_{13,12} = 0.17$, which is due to contamination of modal vectors as modes 12 and 13 are close modes, as discussed above. Here, the WMAC has the advantage over the MAC since, in theory, mass-weighted eigenmodes should be orthogonal and so uncorrelated. That is, $WMAC_{ij} = 0 \forall i \neq j$. As a result, the WMAC can help identify potential sources of error, such as indicating the degree of contamination of EMA-determined eigenmodes due to close modes. This can be confirmed by considering the modes along the diagonal with less than unity correlation. Here, the modes adjacent to the respective mode pairs along the diagonal are correlated by almost the difference from the respective matched mode pair along the diagonal and a unity value correlation. For example, $WMAC_{5,5} = 0.94$ and $WMAC_{5,6} = 0.04$, meaning that EMA estimated eigenmode 5 is 4% contaminated with eigenmode 6, or $WMAC_{13,13} = 0.75$, $WMAC_{13,12} = 0.17$ and $WMAC_{13,14} = 0.05$, meaning that EMA estimated eigenmode 13 is 17% contaminated with mode 12 and 5% contaminated with eigenmode 14. Non-zero off-diagonal terms of modes that are not near to being considered ‘close modes’ may result from the inevitable, but manageable, error associated with numerical methods, such as the reduction and mass-lumping methods used to obtain the mass matrix of the system, or the spatial distribution of the measurement grid resulting in aliasing. For example, the WMAC pairs with indices $\mathbf{S} = \{\{1, 16\}, \{3, 17\}, \{5, 19\}\}$, where $\mathbf{W} = \{WMAC_{i,j} | i, j \in \mathbf{S}\}$. The AutoWMAC of the known orthogonal NMA eigenmodes (the WMAC of the NMA eigenmodes with themselves) for set $\mathbf{S}$ returns $AutoWMAC_{i,j} = 0.04 \forall i, j \in \mathbf{S}$, while the AutoWMAC values for the offset modes close to the diagonal, which are discussed above, return 0. This indicates that there is a small amount of error due to the numerically derived mass matrix of the system and/or spatial sampling. In this case 0.04 of the correlation of the modes in set $\mathbf{W}$ can be attributed to these approximations.

With the correct eigenmode pairing determined, the overall relation of the EMA and NMA eigenfrequencies can be visually compared and analytically quantified. The eigenfrequency values tabulated in Table 4 are plotted with the NMA values against EMA values and the linear regression fit is given in Figure 5. While the linear regression fit quantifies, to an extent, the overall difference for the considered range, the benefit of the visual comparison is that it is not affected by randomness, bias, or artefacts in the data. Visually, the eigenfrequencies in Figure 5 mostly lie along the line $y = x$, indicating an approximate one-to-one agreement. At higher frequencies, the NMA (FEM) eigenfrequencies slightly overestimate the experimental eigenfrequencies, which is confirmed by comparing the plot with its linear regression fit and the PEs in Table 4. The linear regression fit has a slope of 1.02. This indicates that the numerical model is slightly stiffer than the experimental model. The coefficient of determination (R^2) is equal to 1.0, which, when taken into consideration with the plot

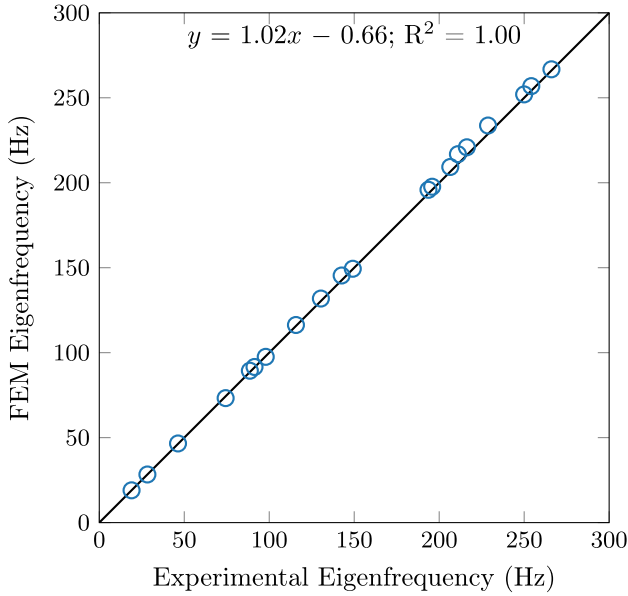


Figure 5. The first twenty paired eigenfrequencies for CLT200a. The line, $y = x$, is plotted for comparison and the linear regression fit with its coefficient of determination is printed.

and linear regression y -intercept term, indicates that the regression coefficient is a good fit. In theory, a perfect agreement between EMA and NMA results in a regression with no intercept term, so the intercept in the linear regression fit can be considered to be the error term of the fit. The error term has a negligible value of -0.66 Hz, indicating that the fit of the regression coefficient is good, requiring a correction of less than 1 Hz. This value satisfies the PE criterion discussed in Section 2.2.1, $PE < 10\%$, for all modes of the system, cf. also Table 4.

Table 5 presents the eigenvalue results of the EMA and NMA for plate CLT200b. Figure 6 presents the EMA-identified eigenmodes. For mode 12, the modal damping ratio convergence criterion had to be relaxed to 0.23 in order for equation (19) to be satisfied. This was likely due to the closeness of mode 12 with mode 13. The eigenfrequency criterion in equation (18) was satisfied for all modes.

Figure 7 presents the WMAC of EMA-fitted eigenmodes against the NMA-determined eigenmodes for plate CLT200b. The general analysis of plate CLT200a presented in this section also applies to CLT200b. The main difference between the plates is the swapping of the order of modes 19 and 20. These modes are almost close modes and so are more prone to crossing over. This can be seen in Table 5, where mode pair 20 agree exactly to three significant figures, $PE_{20} = 0.00\%$, and $PE_{19} = 2.66\%$. Based on the WMAC criteria in Section 2.2.1, 18 of the 20 modes are well correlated with $WMAC \geq 0.9$. Similar to plate CLT200a, the two paired modes that do not meet the criterion of being well correlated are modes 13 and 20, which have values of 0.87 and 0.89, respectively. These values can once again be explained due to close modes as with plate CLT200a. The rest of the non-paired modes are uncorrelated with

Table 5. Eigenvalues for CLT200b. Namely the experimental $f_{e,i}$ and numerical $f_{a,i}$ eigenfrequencies of the i th mode and their percentage error $PE_{i,i}$, plus the experimental modal damping factor $\zeta_{e,i}$ of the i th mode.

i	$f_{e,i}$ (Hz)	$f_{a,i}$ (Hz)	PE_i (%)	$\zeta_{e,i}$ (10^{-2})
1	19.5	19.8	1.54	0.62
2	28.7	28.3	-1.39	0.37
3	47.6	47.8	0.42	0.62
4	74.1	73.2	-1.21	2.00
5	90.9	90.6	-0.33	1.10
6	96.2	99.3	3.22	0.72
7	103	105	1.94	0.59
8	121	124	2.48	0.61
9	132	132	0.00	0.91
10	146	147	0.68	1.00
11	155	156	0.65	0.68
12	199	198	-0.50	1.10
13	201	202	0.50	0.80
14	210	210	0.00	1.10
15	221	233	5.43	0.85
16	226	237	4.87	0.87
17	240	249	3.75	0.92
18	256	257	0.39	0.96
19	264	271*	2.66	0.95
20	267	267*	0.00	1.10

* Eigenmode order corrected.

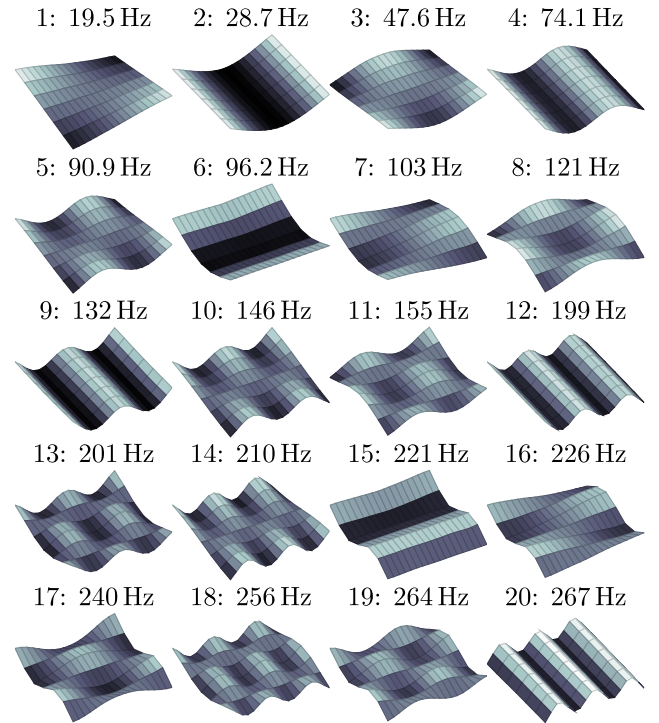


Figure 6. The first twenty identified eigenmodes with their experimental eigenfrequencies for CLT200b.

$WMAC \leq 0.1$. There is just one exception $WMAC_{13,12} = 0.12$, which can be explained due to modal contamination of the EMA-estimated mode 13 with EMA-estimated mode 12 due to the closeness of the modes.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	1	0	0	0	.01	.02	0	0	0	0	0	0	0	0	.02	0	0	0	0	0
2	0	1	0	0	0	0	0	.01	0	0	0	0	0	0	0	0	0	.01	0	0
3	0	0	1	0	0	0	0	.02	0	0	0	0	0	0	0	0	0	.02	0	0
4	0	0	0	1	.99	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.01
5	0	0	0	0	1	.99	0	.01	0	0	0	.01	0	0	0	0	0	0	0	.01
6	.01	0	0	0	0	1	.99	0	0	0	0	0	0	0	.03	0	0	0	0	0
7	.01	0	0	0	0	0	1	.99	0	0	0	0	0	0	0	.03	0	0	0	0
8	0	.01	.01	0	0	0	0	1	.98	.01	0	0	0	0	0	0	.03	0	0	0
9	0	.01	0	0	0	0	0	.01	1	.98	0	0	0	.01	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	.01	1	.99	0	0	.01	0	0	0	0	0	0
11	0	0	.01	.01	.01	0	0	0	.01	1	.99	0	0	0	0	0	0	0	0	.02
12	0	0	0	.01	0	0	0	0	0	0	1	.93	.01	.01	.01	0	0	.01	0	0
13	0	0	0	0	0	0	0	0	.01	.01	0	1	.87	.01	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	.01	.01	1	.99	0	0	0	.01	0	0
15	0	0	0	0	0	.01	.01	0	0	0	0	0	0	1	.97	0	0	0	0	0
16	.05	0	0	0	0	.01	.01	0	0	0	0	0	0	0	1	.97	0	0	0	0
17	0	0	.05	0	0	0	0	.01	0	0	0	0	0	0	0	1	.98	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	0	.01	0	0	1	.98	0	.01
19	0	0	0	0	.04	0	0	.01	0	0	.01	0	0	0	0	0	.01	1	.95	.05
20	0	.01	0	0	0	0	0	0	0	0	0	0	0	0	0	0	.01	.89	.04	.04

Figure 7. WMAC matrix of experimental (vertical) against numerical (horizontal) eigenmodes for CLT200b. The colourmap is a linear scale from white ($WMAC=0$) to black ($WMAC=1$).

With the correct eigenmode pairing for CLT200b determined, the overall relation of the EMA and NMA eigenfrequencies are visually compared and analytically quantified in Figure 8. Visually, the eigenfrequencies in Figure 8 mostly lie along the line $y = x$, indicating an approximate one-to-one agreement. Although at higher frequencies, the NMA eigenfrequencies slightly overestimate the experimental eigenfrequencies, which is confirmed by comparing the plot with its linear regression fit and the PEs in Table 5. The coefficient of determination is equal to 1.0, which, along with the plot and linear regression error term, indicates that the regression coefficient is a good fit. The linear regression fit has a slope of 1.02, as with CLT200a, indicating that the numerical model is slightly stiffer than the experimental model. The error term has a small value of -1.21 Hz, which is twice that for CLT200a. However, the error term still indicates that the fit of the regression coefficient is good, requiring a correction of approximately 1 Hz, which satisfies the PE criterion of $PE < 10\%$ for all modes of the system, cf. also Table 5.

4.2 Harmonic analyses

A frequency-independent loss factor is derived from experiment using the power injection method by averaging the experimentally derived one-third octave band loss factors from 100 Hz to 1600 Hz, see Section 2.1.4. This range selects for one-third octave bands that have a mode count of two or higher and are not influenced by thickness-stretch resonances. The frequency averaged value is presented in Table 3 and used in the subsequent harmonic analyses.

A comparison of the numerical and experimental spatially averaged squared transfer mobilities $L_{|Y|^2}$ is presented

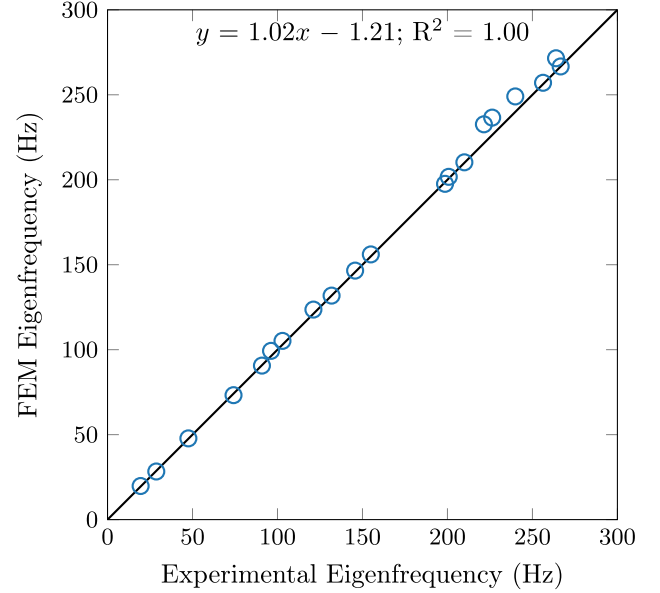


Figure 8. The first twenty paired eigenfrequencies for CLT200b. The line, $y = x$, is plotted for comparison and the linear regression fit with its coefficient of determination is printed.

in Figures 9 and 10 for CLT200a and CLT200b, respectively. The spatially averaged squared transfer mobilities are expressed on a base-10 logarithmic scale $L_{|Y|^2}$ with a reference of $(50 \frac{\text{mms}^{-1}}{\text{N}})^2$, which is a quantity comparable to the sound intensity level of the structure assuming a radiation efficiency of unity [30]. For the experimental results, $L_{|Y|^2}$ is calculated from the “H₁” transfer function. For both CLT200a and CLT200b, there is generally a good overall agreement between the experimental and numerical transfer mobilities for all of the considered excitation points. The largest deviations between the experimental results occur in the low- and high-frequency regions. Specifically below 100 Hz at the second and fourth resonances and in the region of 2500 Hz, where the first thickness-stretch resonance occurs.

The deviations at low frequencies are mainly due to the use of a constant loss factor for the numerical model. A consideration of the damping properties and their effect on the harmonic analyses in this contribution is presented in Figure 11. Figure 11a presents the one-third octave band averaged loss factors derived from the power injection method and modal parameter estimation method, cf. Section 3.4.2. Below 200 Hz, it is seen that the loss factors have larger deviations from the calculated mean. These deviations are quantified in Figure 11b. In Figure 11b, the difference due to the assumption of a constant loss factor is compared to the actual average difference between simulation and experiment at resonance. The one-third octave band differences due to the assumption of a constant loss factor are analytically calculated from the data presented in Figure 11a and the known relation of $|v|^2$ being proportional to $1/\eta^2$ at resonance [17, 21]. The actual difference is calculated by the average of the $L_{|Y|^2}$ differences between

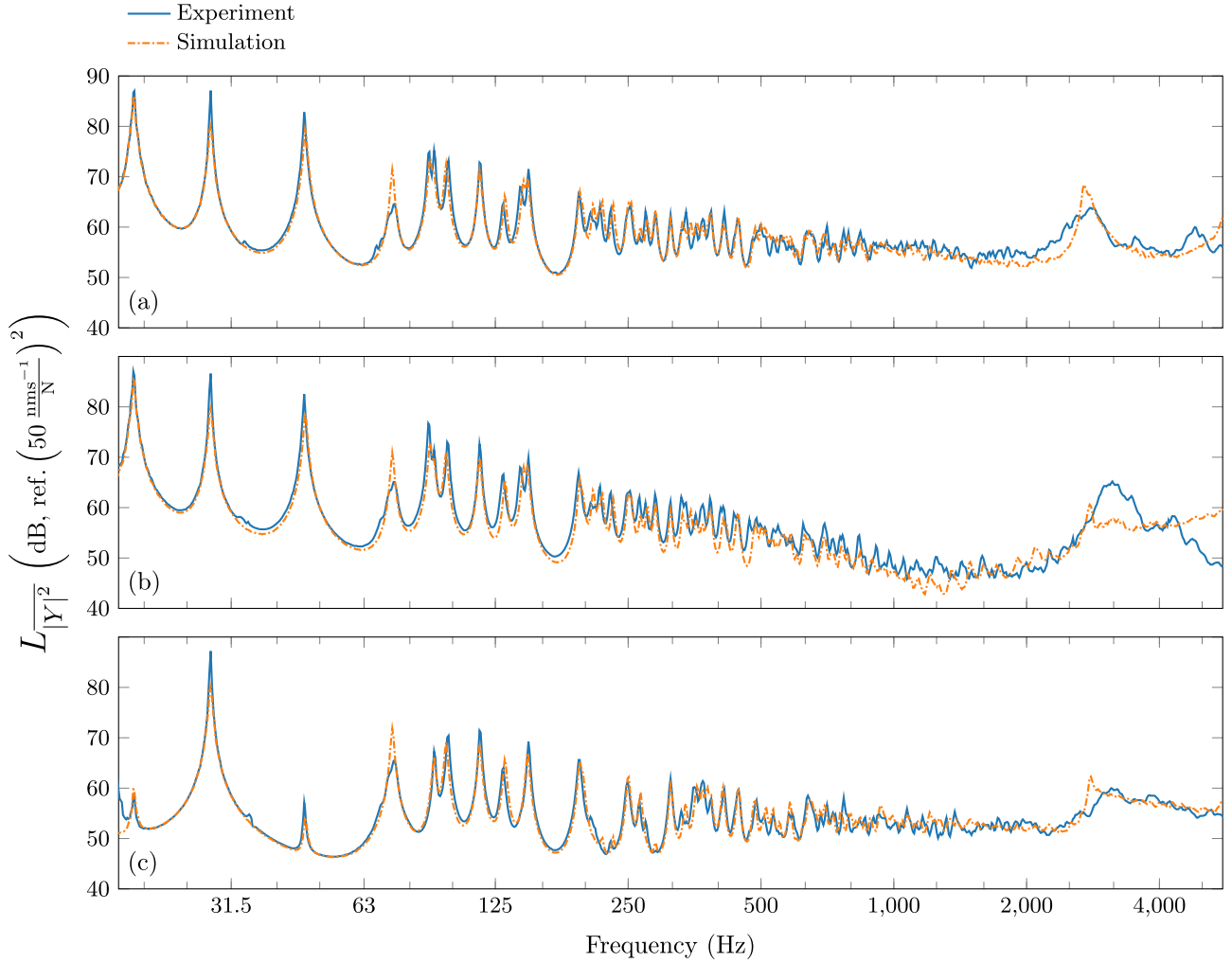


Figure 9. Spatially averaged squared transfer mobility level ($L_{|Y|^2}$) comparison for CLT200a. The experimental and numerical levels are presented on a base-10 logarithmic scale for excitation points (a) F_1 , (b) F_2 , and (c) F_3 .

simulation and experiment at resonance from the data presented in [Figures 9 and 10](#) for the first five resonance peaks. The values for the resonances associated with eigenmodes one (Ψ_1), three (Ψ_3), and five (Ψ_5) are excluded for excitation point F_3 for both of the specimens due to F_3 lying on or near nodes of these eigenmodes and so not exciting these particular eigenmodes efficiently. Comparing the actual $L_{|Y|^2}$ difference and the difference due to the assumption of a constant loss factor, the majority of the differences in the model at low frequencies can be attributed to the assumption of a constant loss factor.

In [Figure 11b](#), it can be seen that the one-third octave loss factors derived from the modal damping ratios are more accurate than the loss factors derived by the power injection method when compared with the actual difference between simulation and experiment. This is due to the limited number of modes in the frequency bands below 200 Hz. For the mid-frequency range, the derived loss factor values appear to converge from the 200 Hz one-third octave band as the number of modes and modal overlap in a given

one-third octave band becomes sufficiently high. It may also be observed that the internal loss factor and modal damping ratios of the plates have an outlier for the nominal 80 Hz one-third octave band. This is due to a combination of the resonance of the air jacks and acoustic radiation of the plate. The critical frequency in the x -direction was $f_{c,x} = 70$ Hz (cf. [Sect. 3.4.2](#)), which corresponds closely with the eigenfrequency of the second bending mode in the x -direction at 74 Hz (cf. [Figs. 3 and 6](#)).

The deviations at higher frequencies in [Figures 9 and 10](#), where the first thickness-stretch resonance cuts on, may also be partially due to the use of a constant loss factor. However, a source of difference would also lie in the method used to calculate the loss factor presented in [Section 2.1.4](#). This would be due to the calculation of the maximum strain energy of the system being approximated as proportional to the spatial average of the square of the surface velocity. This assumption only holds when the infinitesimal elements through the thickness of the plates are moving uniformly, that is, when no thickness-stretch modes are present.

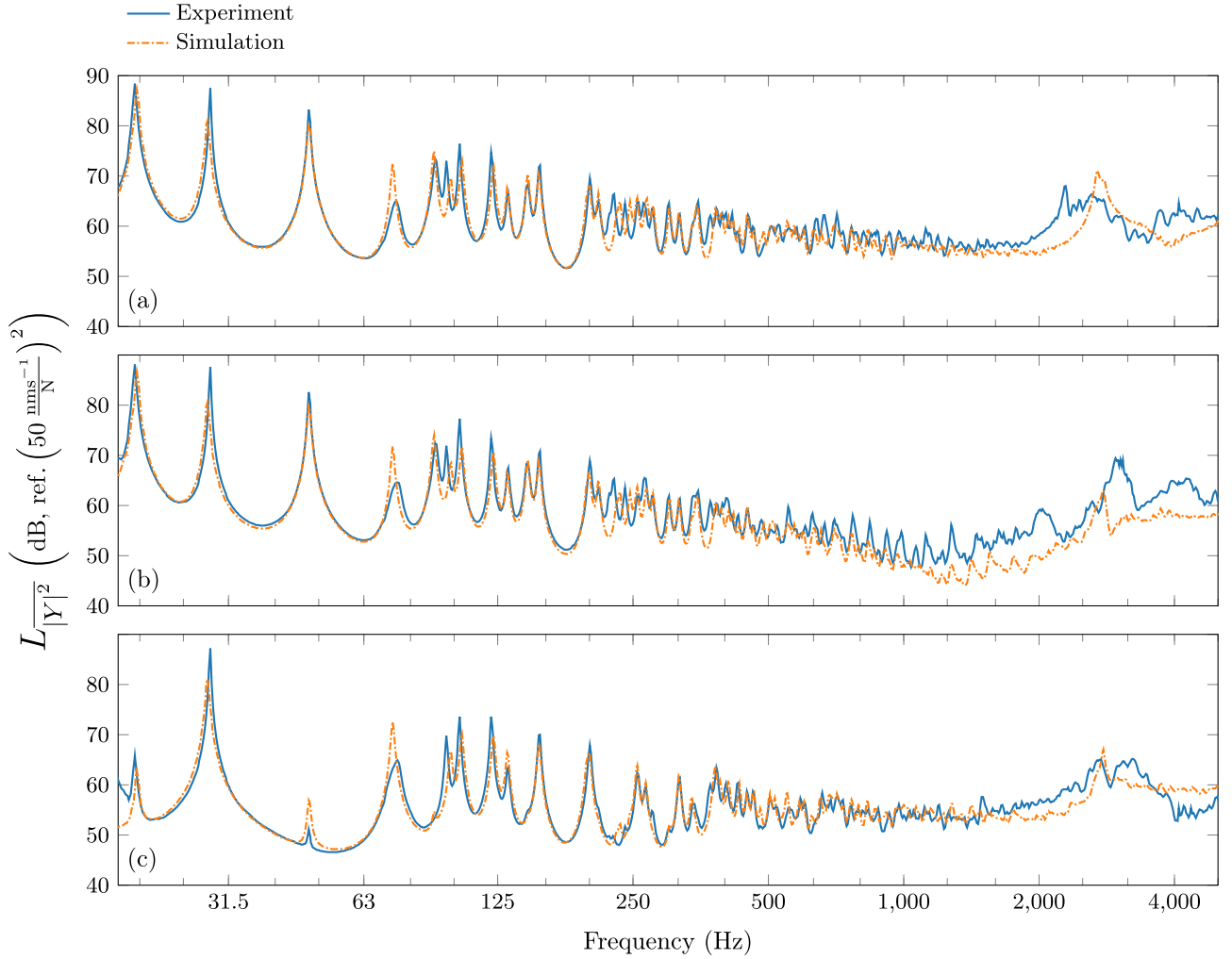


Figure 10. Spatially averaged squared transfer mobility level $\left(L_{|Y|^2}\right)$ comparison for CLT200b. The experimental and numerical levels are presented on a base-10 logarithmic scale for excitation points (a) F_1 , (b) F_2 , and (c) F_3 .

However, other sources of error, such as the global mesh size being limited by the available computational resources and uncertainties in the measurement results, as the measurement set-up was optimized for the lower frequency range, are likely more significant sources of error. It should be noted that for these high-frequency resonances, it is difficult, if possible at all, to extract the modal damping ratios by means of EMA due to the high modal overlap.

Another observable detail from the results presented in Figures 9 and 10 is that excitation point F_2 , located in the vicinity of the half-lap joint notch, has the worst agreement of the three excitation points for both of the plates. This is likely because the notches in each of the plates were not modelled with the plate instead having a constant thickness. This idea supported by the larger deviations at higher frequencies, likely as the wavelength is approaching the size of the notch.

Overall, Figures 9 and 10 show a good general agreement between the ESL CLT model and experimental results. A frequency-dependent loss factor in the model

could account for the differences found below 200 Hz. Differences found at high frequencies may not be due to the assumption of a frequency-independent loss factor, but rather the measurement set-up and mesh size of the numerical model. It was found that the fitted modal damping ratios are more accurate for determining the loss factor of the plates than the energy-based power injection method for the low-frequency range. The values appear to converge for the mid-frequency range, while for the high-frequency range, the power-injection method is the most appropriate method to determine the loss factors.

5 Conclusion

This contribution compares and correlates experimentally estimated and numerically calculated higher-order modal parameters for CLT plates. Despite being a composite structural element, evidence is presented that CLT elements consist of real eigenmodes, and so are

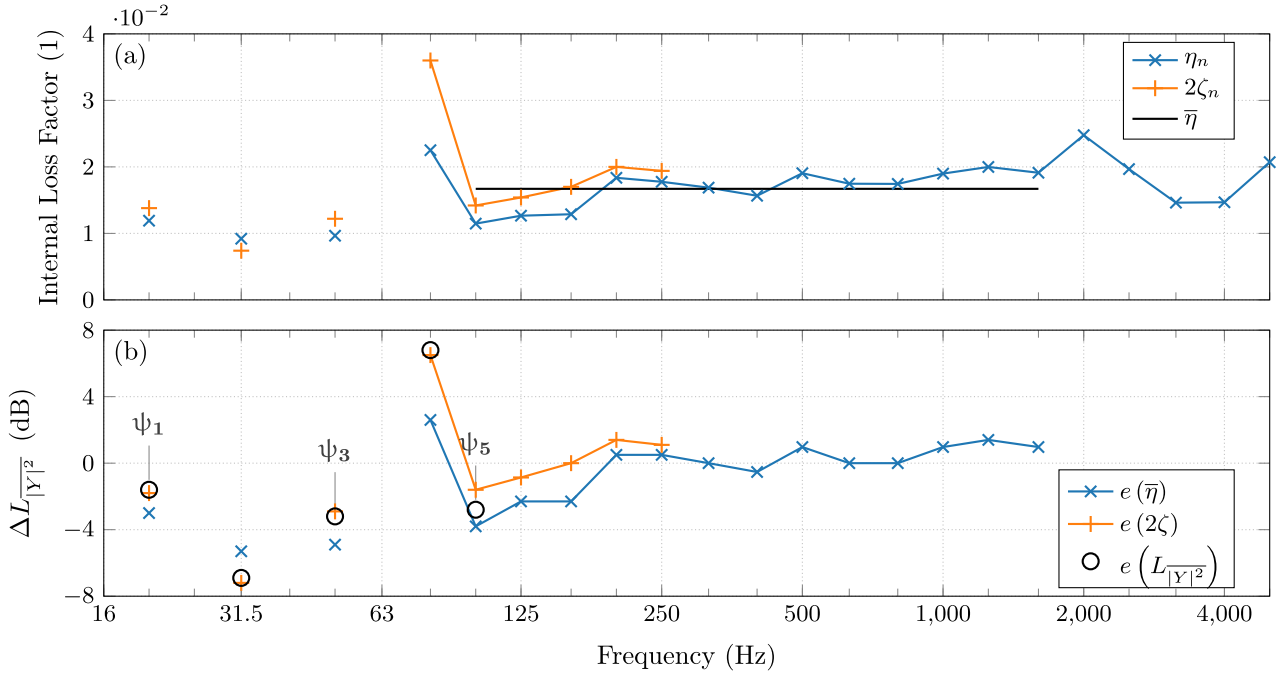


Figure 11. Consideration of damping properties. (a) Average internal loss factors on a one-third octave band basis. η_n : Power Injection Method (PIM). $2\zeta_n$: EMA modal damping ratios. $\bar{\eta}$: PIM loss factor averaged from 100 Hz to 1600 Hz, $\bar{\eta} = 0.0167$. (b) Average one-third octave band transfer mobility difference, $\Delta L_{|Y|^2}$. $e(\bar{\eta})$: Difference in simulation when using one-third octave band loss factor instead of $\bar{\eta}$. $e(2\zeta)$: Difference in simulation when using modal loss factors instead of $\bar{\eta}$. $e(L_{|Y|^2})$: Difference between experiment and simulation for the first five eigenmodes.

proportionally damped. Any complexity of the modes was found to be due to close modes. This could be determined due to a mass-normalised correlation between numerically calculated real modes and the experimentally estimated modes. The mass-normalisation procedure utilised in this contribution ensured the orthogonality of the eigenmodes so that spurious correlations between modes were removed, allowing for differences and similarities among eigenmodes to be identified.

The complementary use of modal and response models is demonstrated for the verification and validation of theoretical models for vibro-acoustic problems. The modal domain with mass-normalised shape-based and scalar-based indices can provide an orthogonal basis and, therefore, a mathematically unique solution for predicting the structural response of CLT elements. However, the frequency domain (response model) with an energy-based index is still required to understand the complete behaviour of CLT, such as thickness-stretch resonant modes that are nascent at higher frequencies. It was found that in the low-frequency range, where the mode count and modal overlap are low, the damping properties are more accurately derived from modal parameter estimation rather than the energy-based response of the structure. The results of the methods were found to converge for the mid-frequency range.

This contribution also presents further evidence of the adequacy of homogenised models for the vibro-acoustic analysis of cross-laminated timber elements. ESL models whose properties were derived from layerwise models were verified and validated in both the modal and frequency

domains, with key indices and criteria being considered as part of the validation process.

An extension of this work would be the consideration of another energy-based index to quantify the difference between response models. In addition, an appropriate criterion should be considered as well. Also, consideration of another curve fitting algorithm, such as the polyreference time domain method, to overcome the limitations of the least squares complex exponential method used in this contribution with close modes. The next step in improving the accuracy of the response model would be to implement different loss factor values over two or potentially three frequency ranges. These regions could be divided up based on modal density and when the first thickness-stretch eigenmodes cut-on. Lastly, the CLT elements considered in this contribution have glued edges. The effects of edge gluing in CLT elements, particularly on damping properties, would also be a point of further investigation.

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Conflicts of interest

The authors declare no conflicts of interest

Data availability statement

Data are available on request from the authors

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