



Numerical study on role of foot of a flue organ pipe: relative phases in oscillations among pipe, foot and jet

Shiryu Ikoga¹, Tatsuki Onomata¹, Ryoya Tabata¹, Sho Iwagami¹, Taizo Kobayashi^{2,3},
and Kin'ya Takahashi^{1,2,3,*} 

¹ Department of Physics and Information Engineering, Kyushu Institute of Technology, 680-4 Kawazu, Iizuka, 820-8502, Japan

² Research Institute for Information Technology, Kyushu University, 744 Motooka, Nishi-ku, Fukuoka, 819-0395, Japan

³ Acsioma Ltd., 3-8-33 Momochi Hama, Sawara-ku, Fukuoka, 814-0001, Japan

Received 1 April 2024, Accepted 5 September 2024

Abstract – Recent numerical studies showed that the foot of flue organ pipes closed at one end acts as a Helmholtz resonator and influences the acoustic oscillation in the pipe. Namely, the relative phase of oscillations between the pipe and foot changes depending on the resonance frequency of the foot. This fact can be explained by the theory of forced harmonic oscillators, in which the pipe works as a force driving the foot. However, in the damping-controlled regime, in which the resonance frequency of the foot, f_H , is nearly equal to the frequency of acoustic oscillation, f_a , the amplitude of the acoustic oscillation is smaller than those in the mass-controlled regime $f_H < f_a$ and the stiffness-controlled regime $f_H > f_a$, even though the pressure oscillation in the foot lags behind that in the pipe by $\pi/2$. In this paper, to investigate this problem and to explore the role of the foot in detail, using a three-dimensional model of a closed flue organ pipe, we numerically investigate the relative phases among the pressure oscillation in the pipe, that in the foot and the jet velocity. We conclude that total balance among them is crucial for sustaining acoustic and fluid oscillations in the sound generation process. The oscillating jet drives the pipe with appropriate timing to sustain the acoustic oscillation for the stiffness-controlled and mass-controlled regimes. In contrast, the jet inefficiently drives the pipe with inappropriate timing for the damping-controlled regime.

Keywords: Flue organ pipe, Foot, Helmholtz resonator, Aeroacoustic simulation

1 Introduction

Studying the sounding mechanism of air-jet instruments (flue instruments), such as flue organ pipes, recorders and flutes, is one of the long-standing problems of musical acoustics [1–3]. Many aspects of the sounding mechanism have been clarified, such as the relationship of the acoustic frequency of the pipe with the jet velocity, the details of the attack transient, the behavior of the jet motion changing with the jet velocity as well as the geometry of the mouth opening, and the energy transition between the fluid and acoustic fields near the mouth opening [3–11]. Concerning flue organ pipes, there are several crucial points for understanding the acoustic and hydrodynamic properties in the sound generation process [3, 10, 11]. As an example in the problem of controllability of instruments, the influence of the window way, e.g., flue length and chamfers, has been studied by several authors [3, 9]. Ségoufin *et al.* experimentally investigated the influence of the geometry of the flue with or without chamfers on the jet motion and acoustic

oscillation for a recorder-like flue organ pipe [9]. Their result indicated that shortening the flue allows better control of the instrument and makes the sound spectrum richer in higher harmonics however adding chamfers to the flue effectively stabilizes oscillation only for a long flue but not for a short flue.

Here, we focus on the role of the foot in the sound generation process. The influence of the foot geometry on the jet motion and acoustic oscillation has seemed to attract less attention, although as a related issue, the effect of the vocal tract on the sound production of recorders was reported by several authors [12, 13]. In the previous work [14], we found with numerical simulation of two-dimensional (2D) models that the foot acts as a Helmholtz resonator and the change of its resonance frequency affects the stability of the acoustic oscillation and the relative phase between the pressure oscillation in the pipe and that in the foot. Such a change of the relative phase was explained by the theory of forced harmonic oscillators (TFHO), in which we assumed that the pipe works as a force driving the foot. Indeed, in the case that the Helmholtz resonance frequency f_H of the foot is sufficiently smaller than the acoustic

*Corresponding author: takahasi@acsioma.jp

oscillation frequency f_a of the pipe, the pressure in the foot oscillates in anti-phase with that in the pipe. In the resonance condition of $f_H \approx f_a$, the oscillation in the foot lags behind that in the pipe by nearly $\pi/2$ but both oscillations become relatively smaller in amplitude and considerably unstable. Next, to explore the reason for this unexpected fact, we investigated relative phases among the oscillation in the foot, that in the pipe and the jet motion. However, we did not find clear evidence owing to an inherent instability of 2D fluid, e.g., inverse energy cascade [10, 15]. In the study on three-dimensional (3D) models [16], we obtained essentially the same result: the relative phases between the oscillations in the foot and pipe changes depending on f_H . In the resonance condition of $f_H \approx f_a$, the oscillation in the pipe becomes relatively smaller in amplitude but almost stable because acoustic oscillations and fluid motions are more stable in 3D systems than in 2D systems [10, 11]. Therefore, the 3D simulation provides more realistic and reasonable results and is suitable to explore the influence of the foot on the sound generation mechanism taking into account the nonlinear interaction between the jet motion and the acoustic oscillations.

In this paper, we consider that the study of relative phases among the acoustic oscillation of the foot, that in the pipe and jet velocity is the key to understanding the change of the acoustic oscillations depending on the Helmholtz resonance frequency of the foot. Using the same 3D model as the previous work [16], we numerically investigate the relative phases in detail for the stiffness-controlled, damping-controlled, and mass-controlled regimes, in particular focusing on the damping-controlled regime with $f_H \approx f_a$, in which relatively smaller oscillations are observed. The horizontal jet velocity v_x at the flue exit is periodically perturbed by the acoustic oscillation in the foot and oscillates in different phases with the vertical velocity of the jet v_y . We investigate the problem of how the change of the relative phase between v_x and v_y affects the process for the jet to drive the pipe. Furthermore, with the help of a negative-displacement jet model [17–22], we estimate the relative timing of injecting the maximum velocity flow to the pipe in the area near the edge because according to recent studies based on the Howe’s energy corollary, the acoustic energy is majorly generated in this area [11, 23–26]. We show that consistency in relative phases among v_x , v_y , the acoustic oscillation in the pipe and that in the foot, i.e., total balance among them, plays a crucial role in the sound generation process.

2 Model and numerical setting

In this paper, we study a model system similar to the flue organ pipe experimentally studied by Ségoufin *et al.* [9]. However, to reduce the mesh size, namely the computational task, our model takes a closed pipe with a length of $L = 141.5$ mm so that its fundamental pitch is almost the same as the open pipe with a length of 283 mm studied by Ségoufin *et al.* [9, 14, 16]. The models studied in this paper are the same as those studied in the previous work [16]. Note that, as shown in the previous works [10, 11], a

model with a closed pipe has essentially the same fundamental features as air-jet instruments with an open pipe, even though some non-negligible differences between open and closed pipes were reported by several authors [27–30]. Figures 1a and 1b show a three-dimensional image of the model called “Reference model” and dimensions in an area of the mouth opening, respectively. The model consists of an inlet (tube), a foot, a flue and a pipe. The volume labeled “Outside” connected to the mouth opening indicates an outside space with the external atmosphere and its dimensions are the same as those in the previous works [11]. The flue is chamfered at its exit because the flue with chamfers rather stabilizes the jet oscillation and acoustic oscillation [9, 14]. The distance between the flue exit and the edge is set as $l = 4$ mm. The cross-section of the pipe is a square: the height $h_p = 20$ mm and width $w_p = 20$ mm. By using the method introduced by Verge *et al.* [5], the end correction at the mouth opening is given as $\Delta L_{\text{pipe}} = 39.05$ mm, and the fundamental frequency of the closed pipe is estimated as 481.3 Hz.

As shown in Figure 2, we prepare three models called the “Reference model”, “Mid model” and “Short model”, which are different in foot geometry. The foot consists of a rectangular parallelepiped and a tapered channel connecting with a flue, whose underside is formed by a quarter-cylinder block. The rectangular part of the Reference model is the same as that of the instrument studied by Ségoufin *et al.* [9] and Ikoga *et al.* [16]: its length, height and width are taken as $l_{cb} = 60$ mm, $h_{cb} = 20.8$ mm and $w_{cb} = 20$ mm, respectively. This quarter-cylinder block with a radius of $R = 19.8$ mm is almost the same as that in the experiment, while the length of the flue, $l_f = 3$ mm, is shorter than that in the experiment, i.e., 15 mm, [9]. This is because, in the previous study of 2D models [14], a model with a flue of $l_f = 3$ mm length achieved a shorter attack transient and was slightly more stable in the stationary state than a model with $l_f = 15$ mm. The other two models are different in length of the rectangular part: the “Short model” with a length of 30 mm and the “Mid model” with a length of 45 mm. Since the foot acts as the Helmholtz resonator, the three models are different in the foot resonance frequency as will be shown in Section 3.1.

Aeroacoustic simulation is necessary for studying air-jet instruments because the sound generation process is described in terms of mutual interactions between the acoustic and fluid fields [10, 11]. In this paper, we utilize a compressible Large Eddy Simulation (LES) solver with a one-equation eddy viscosity subgrid-scale model: the rhoPimpleFoam in the OpenFOAM Ver.5.0 and v2006. To achieve enough accuracy, the minimum grid size is set as $\Delta x = 0.1$ mm in an area of the mouth opening, the number of cells is taken as 165,312,887 for the Reference model, and the time step is set as $\Delta t = 5.0 \times 10^{-8}$ s [16]. The temperature and pressure in equilibrium are set as $T_0 = 300$ K and $p_0 = 1.0 \times 10^5$ Pa, respectively. So, the pressure fluctuation is defined as $p - p_0$. The average velocity in the flue is set at 6 m/s in the steady state. As shown in the previous works [14, 16], acoustic oscillations are well sustained in the pipe and foot at this flow velocity.

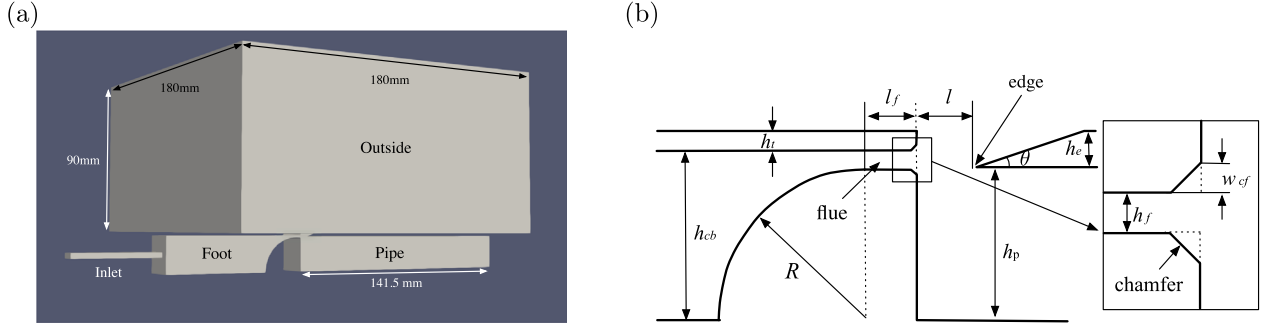


Figure 1. Geometry of a flue organ pipe with a closed pipe of 141.5 mm in length (Reference model). (a) 3D view of the model with an outer region. The length, height and width of the inlet are taken as $l_{in} = 50$ mm, $h_{in} = 3$ mm and $w_{in} = 10$ mm, respectively. (b) Dimensions of the edge, flue with chamfers and foot channel on the 2D cross-section: $h_p = 20$ mm, $l = 4$ mm, $\theta = 20^\circ$, $w_{cf} = 0.71$ mm, $h_f = 1$ mm, $l_f = 3$ mm, $h_{cb} = 20.8$ mm, $R = 19.8$ mm, $h_e = 2.51$ mm and $h_t = 1.71$ mm; the widths of the pipe and foot are taken as $w_p = 20$ mm and $w_{cb} = 20$ mm, respectively.

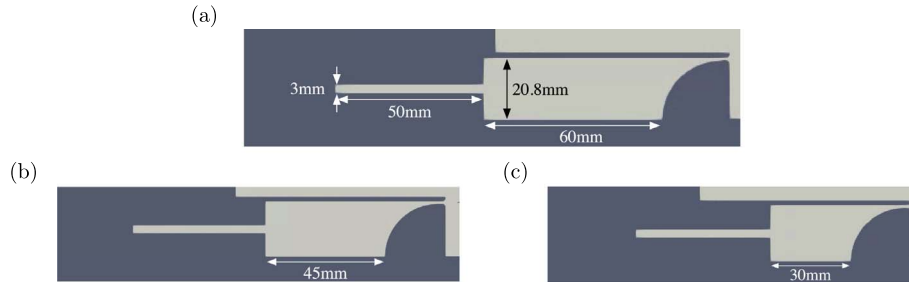


Figure 2. Geometry of three foot models. (a) Reference model. (b) Mid model. (c) Short model.

A uniform flow is injected from the entrance of the inlet and the flow speed takes a value of 4 m/s in the steady state, since its cross-section, $3 \times 10 \text{ mm}^2$ (see the caption of Fig. 1), is 1.5 times larger than that of the flue, $1 \times 20 \text{ mm}^2$. To avoid an initial shock wave, the flow speed is gradually increased until $t = 2$ ms to reach a uniform flow with a velocity of 4 m/s. The top and side walls of the Outside are set as the wave transmissive outflow condition of the OpenFOAM, which approximates the Navier-Stokes characteristic boundary conditions [31]. As shown in the previous work [11], this setting of the Outside with enough volume almost completely absorbs outgoing flow and acoustic waves; thus, resonance in the “outside” is not an issue. The other walls of the model are set as non-slip solid walls [10]. Note that in OpenFOAM the inlet boundary, i.e., the entrance cross-section of the inlet, behaves as a diode, that is, it becomes a transparent wall for an incoming flow but a solid wall for an outgoing flow [14]. We calculate the models with the supercomputer, Fujitsu PRIMERGY CX2550/CX2560 M4 (3456 GFLOPS/node). For the cases of the Reference model, the calculation of 40 ms using 32 nodes with 36 cores, namely 1152 parallel computing, takes about 600 h.

3 Numerical result

3.1 Resonance frequencies of the three foot models

First, we determine the Helmholtz resonance frequencies of the three foot models detached from the pipe. A Helmholtz resonator conceptionally consists of the body

and neck. The length and cross-section of the neck and the volume of the body are necessary to calculate the resonance frequency using the theoretical formula [1]. An effective body of the foot should consist of the inlet, the rectangular part, and a part of the channel between the rectangular part and the flue, while an effective neck should be formed by the flue and the remainder of the channel. However, it is difficult to determine the boundary between the body and neck owing to its intricate shape. Thus, we numerically estimate the resonance frequency of the foot without the pipe using the compressible LES [14]. As shown in Figure 3a, we construct a Helmholtz resonator directly connected to an outside space and drive it to estimate resonance frequency by injecting into the inlet a periodically changing flow as $U_{in}(t) = U_0 \sin \omega t$ with $U_0 = 1$ m/s, where driving frequency $f = \omega/2\pi$ is taken at intervals of 10 Hz and obtained values include the ambiguity of ± 5 Hz (for details, see the Supplemental material, §1).

Figure 3a shows a pressure field of the resonance state for the Reference foot model. Figure 3b shows pressure fluctuations $p - p_0$ of the resonance states observed at the point A (see Fig. 3a) for the three foot models together with that for the Reference foot model at the off-resonance condition of $f = 480$ Hz. Note that the point A is set at the upper-left side of the rectangular part at which an acoustic pressure wave can take the maximum value in amplitude far from its nodes owing to the Neumann boundary condition imposed on the pressure field, and disturbances induced by fluid flow are almost ignored. Table 1 shows the

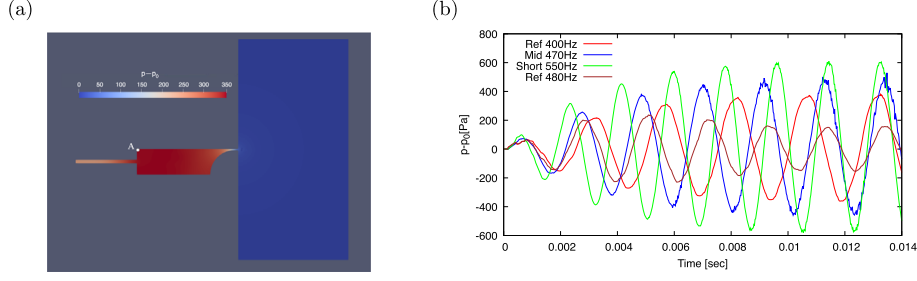


Figure 3. Pressure in the Helmholtz resonator. (a) A snapshot of the distribution of pressure fluctuation $p - p_0$ at $f = 400$ Hz for the Reference foot model. The point A is an observation point at the center of the top-left edge of the rectangular parallelepiped. (b) Pressure fluctuations $p - p_0$ at resonance observed at the point A for the three foot models, and that for the Reference foot model at an off-resonance condition of $f = 480$ Hz.

Helmholtz resonance frequencies f_H obtained for the three foot models compared with the frequencies of acoustic oscillations in the pipe, f_a , which will be discussed in Section 3.2. Then, f_H seems to decrease with the volume of the effective body of the Helmholtz resonator V_H as $\propto 1/\sqrt{V_H}$ [1], where V_H can be roughly estimated by the sum of the volumes of the rectangular part and inlet. For the Reference foot model, the pressure amplitude at $f = 480$ Hz $\approx f_a$ is only 44% that at $f = f_H = 400$ Hz. Namely, the amplitude at $f = 480$ Hz is more than 6 dB smaller than that at $f_H = 400$ Hz. Thus, the oscillation at $f = 480$ Hz is out of resonance (see also the Supplemental material, §2). Note that the oscillation for the Mid foot model is disturbed by high-frequency noise, which should be generated by high-frequency modes excited by the nonlinear interaction of the fluid with acoustic modes owing to a certain geometry nature of the Mid foot model as discussed in the Supplemental material, §1.6.

3.2 Pressure fluctuations in the pipe and foot for the three models

Figure 4 shows the snapshots of spatial distributions of velocity and pressure at $t = 38.74$ ms in the stationary state for the Reference model. As shown in Figure 4a, the jet spontaneously oscillates colliding with the edge and emits aerodynamic sound to drive the resonance pipe. The flow emanating from the inlet diffuses in the middle of the rectangular part of the foot (see also the Supplemental material, §3). As shown in Figure 4b, the acoustic pressure oscillations in the pipe and foot are well sustained, but the pressure in the foot oscillates in anti-phase with that in the pipe, as reported in the previous works [14–16].

Figure 5 shows pressure fluctuations, $p_f - p_0$ and $p_p - p_0$, observed at the points A and B in Figure 4b together with the averaged jet velocities $\bar{v}_x$ and $\bar{v}_y$ at the flue exit, which will be discussed in Section 3.3, for the three models in the left column, and Fourier spectra of these oscillations in the range ($20 \leq t \leq 40$ ms) in the right column. For all the models, stable pressure oscillations are observed in the stationary state, and their frequencies f_a are given in Table 1. Since the calculation time is not long enough to attain an acceptable resolution in frequency by the Fourier transformation, we estimate the frequency from the mean

Table 1. Helmholtz resonance frequencies f_H and acoustic oscillation frequencies f_a .

Model	f_H [Hz]	f_a [Hz]	$f_a - f_H$ [Hz]
Reference	400 ± 5	481	81 ± 5
Mid	470 ± 5	469	-1 ± 5
Short	550 ± 5	461	-89 ± 5

period in the stationary state after $t = 20$ ms (see the Supplemental material, §1.3). The frequency of the acoustic oscillation f_a for the Reference model is around 481 Hz, which is almost the same as the theoretical prediction of 481.3 Hz obtained in Section 2. However, as shown in Table 1, the frequencies f_a are slightly smaller than the theoretical prediction for the Mid and Short models. This is mainly because when the resonance frequencies of the separated pipe and foot are close to each other, the resonance frequencies for the combined system of the pipe and foot considerably shift from the original frequencies, and also because the interaction with the nonlinear element, i.e., the oscillating jet, influences the resonance conditions (see the Supplemental material, §1.7). Note that for every model a large pressure peak is observed in the foot during the attacking period $0 < t < 2$ ms owing to the increasing volume flow from the inlet (see the Supplemental material, §3).

For the Reference model, the pressure in the foot oscillates nearly in anti-phase with that in the pipe after a short attack transient. The amplitude of oscillation in the foot is almost half of that in the pipe in the stationary state. On the other hand, the Short model has a relatively long attack transient, and the pressure in the foot oscillates nearly in phase with that in the pipe. The amplitude of oscillation in the foot is almost half of that in the pipe. For both models, the pressure oscillations in the pipe are very stable and large in amplitude.

Let us consider the Fourier spectra of pressure oscillations in the foot and pipe labeled “foot p_f ” and “pipe p_p ”. For the Reference model, the fundamental peaks labeled “ f_a ” are well formed and the small 3rd harmonic peaks “ $3f_a$ ” are observed as a characteristic of the closed pipe.

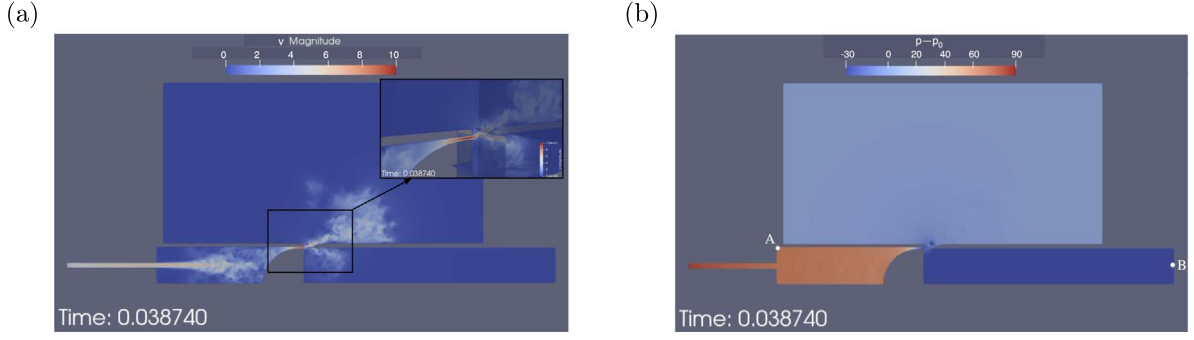


Figure 4. Distributions of the magnitude of velocity v and pressure fluctuation $p - p_0$ at $t = 0.03874$ s. (a) Magnitude of velocity v (b) Pressure fluctuation $p - p_0$. The points A (the same as in Fig. 3) and B (the center of the right end of the pipe) are observation points of pressure in the foot and pipe, respectively.

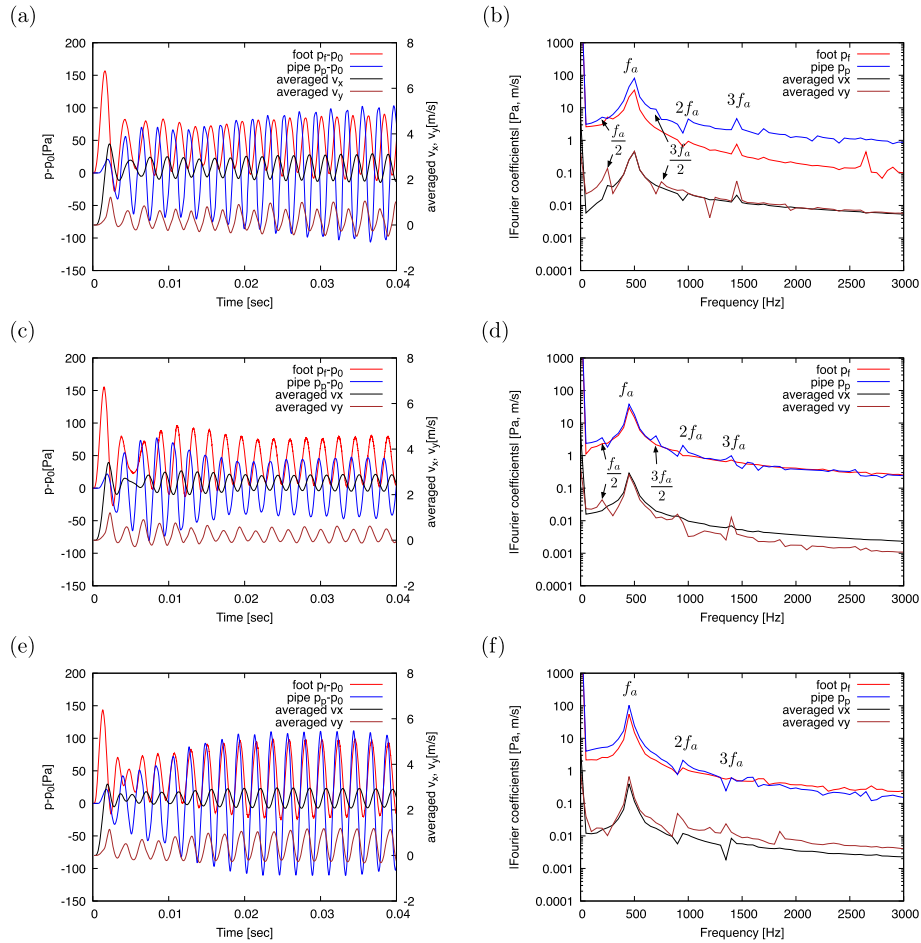


Figure 5. Pressure fluctuations in the foot $p_f - p_0$ and those in the pipe $p_p - p_0$ together with averaged velocities $\bar{v}_x$ and $\bar{v}_y$ at the flue exit. (a) Oscillations for the Reference model. (b) Fourier spectra of the oscillations in the range $(0.02 \leq t \leq 0.04)$ s for the Reference model. (c) Oscillations for the Mid model. (d) Fourier spectra for the Mid model. (e) Oscillations for the Short model. (f) Fourier spectra for the Short model.

In contrast, the tiny 2nd harmonic peaks “ $2f_a$ ” appear due to nonlinear interaction with the fluid flow. The sub-harmonic peaks labeled “ $\frac{f_a}{2}$ ” and “ $\frac{3f_a}{2}$ ” appear only for the oscillation in the pipe. This issue will be investigated relating to the jet motion in Section 3.3. A peak observed around 2700 Hz for the oscillation in the foot is regarded as one

of the high-frequency modes of the foot as shown in the Supplemental material, §1.3–1.5. Concerning the Fourier spectra for the Short model, the fundamental peaks “ f_a ” and the tiny 2nd harmonic peaks “ $2f_a$ ” appear, while clear peaks of the 3rd harmonic “ $3f_a$ ” are not observed. Furthermore, there are no clear sub-harmonic peaks.

For the Mid model, the pressure oscillations in the pipe and foot are still stable in the stationary state. However, the oscillation in the foot is slightly smaller than those for the other models. The oscillation in the pipe leads that in the foot by nearly $\pi/2$, and is almost the same in amplitude as that in the foot; thus, its amplitude is nearly half of the amplitude of the oscillations in the pipe for the other models. The oscillations in the pipe and foot behave in a slightly complicated manner in a relatively long attack transient. Indeed, except for the first large peak in the foot, the oscillation in the pipe first grows, and that in the foot follows. Then, they slightly reduce towards the stationary state. From these facts, the Mid model is less stable than the other two models. Concerning the Fourier spectra, the fundamental peaks “ f_a ” appear for the pressure oscillations in the foot and pipe. However, the peaks of the 2nd and 3rd harmonics, “ $2f_a$ ” and “ $3f_a$ ”, appear only for the oscillation in the pipe. The sub-harmonic peaks labeled “ $\frac{f_a}{2}$ ” and “ $\frac{3f_a}{2}$ ” appear for the oscillation in the pipe.

Therefore, the phase difference between the pressure oscillation in the pipe and that in the foot changes depending on the models, namely the volumes of the foot. As shown in the previous works [14–16], the change in the phase difference can be explained by relying on the theory of forced harmonic oscillators (TFHO) [14]. Namely, we can assume that the acoustic oscillation at $f = f_a$ in the pipe drives the foot of the resonance frequency f_H . Thus, the Short, Mid and Reference models are considered to be in the stiffness-controlled ($f_a < f_H$), damping-controlled ($f_a \approx f_H$), and mass-controlled ($f_a > f_H$) regimes, respectively.

According to TFHO, the oscillation in the foot should take the maximum amplitude for the Mid model, because it is in resonance with that in the pipe. However, this is not the case. Furthermore, the oscillation in the pipe, i.e., the driving force, is quite smaller than those for the other two models. We can consider that the enhancement of energy transfer from the pipe to the foot owing to resonance consumes the acoustic energy in the pipe as a sort of Helmholtz silencer. As an alternative interpretation, the system of the pipe and foot may be regarded as coupled linear oscillators. Indeed, using coupled damped oscillators with the same resonance frequencies driven by a periodic force, which is the oscillating jet in this case, we can interpret the complicated oscillations in the attack transient as alternate energy transfer between the pipe and foot and also explain that the damping force, e.g., the fluid viscosity, makes the system soon converge to the stationary oscillation, though the relative phase between the oscillation in the pipe and that in the foot is not reproduced (see Supplemental material, §1.7). Unfortunately, these interpretations cannot explain the reduction of the total acoustic energy in the pipe and foot for the Mid model. We consider how the jet drives the pipe as the key to solving this problem. In the following subsections, we focus on the jet motion, namely the jet velocity of the flue exit, and investigate relative phases among the pressure oscillations in the pipe and foot, and the jet velocity [15].

Note that, for the 2D models studied in the previous work [14], the resonance frequency of the foot is smaller

than that of a corresponding 3D model due to reasons discussed in the Supplemental material, §2. In the previous work [14], we did not study a model corresponding to the 3D Mid model because a near resonance condition $f_a \approx f_H$ is satisfied for the 2D Short model, which corresponds to the 3D Mid model.

3.3 Jet velocity at the flue exit

In this subsection, we consider the motion of the jet at the flue exit as shown in Figure 6a, because the jet motion is considerably affected by the acoustic oscillations in the foot and pipe. Hereafter, we take the origin of the coordinate system at the center of the flue exit: the x -coordinate is taken in the longitudinal direction of the pipe, the y -coordinate is in the vertical direction and the z -coordinate is in the spanwise direction.

Figure 6b shows the fluctuations of the x component of the velocity, v_x , observed at the center of the flue exit, namely the point C in Figure 6a, for the three models. For the Reference model, v_x stably oscillates with a large amplitude at the same frequency as the pressure fluctuations in the pipe and foot, although it rather behaves as a subharmonic oscillation in the steady state, which will be discussed later. For the Mid model, v_x oscillates with a relatively small amplitude, which undulates gradually. For the Short model, v_x oscillates a bit irregularly with a relatively small amplitude. As shown in the inset, the flow emanating from the exit of the inlet collides with the quarter cylinder block, goes along it, and enters the flue. Thus, it disturbs the jet velocity. Note that for all the models, the time averages of v_x take values around 8 m/s, which are a little larger than the average flow velocity of 6 m/s in the flue. Such an enhancement of velocity at the center of a jet is often observed, in particular for bell-shaped jets.

To investigate the relative phases of the jet velocity to the pressure oscillations in the pipe and foot, we need data on regularly oscillating velocities. To do this, we take the averages of the x and y components of the velocity over the cross-section of the flue exit, namely the sampling plane in Figure 6a. As shown in Figure 5, the averaged velocity components $\bar{v}_x$ and $\bar{v}_y$ regularly oscillate for all the models including the Short model. For the three models, every $\bar{v}_x$ oscillates around a value of 2.5 m/s. This is because the area of the sampling plane, $20 \times 2.42 \text{ mm}^2$, is 2.42 times larger than the cross-section of the flue, $20 \times 1.0 \text{ mm}^2$, and $\bar{v}_x \approx 2.5 \text{ m/s}$ times 2.42 is nearly equal to the averaged flow velocity of 6 m/s in the flue. The oscillating amplitude of $\bar{v}_x$ takes the largest value for the Reference model and the smallest value for the Mid model because it is induced by the difference of the pressure in the foot p_f from that at the mouth opening ($\approx p_0$).

For the Reference model, $\bar{v}_y$ oscillates taking positive and negative values, and behaves as a subharmonic oscillation in the steady state. Its time average is larger than zero and such a net outgoing flow from the mouth opening is expected for closed pipes [11]. In this case, $\bar{v}_x$ is not synchronized with $\bar{v}_y$, i.e., nearly anti-phase synchronization between $\bar{v}_v$ and $\bar{v}_y$. For the Mid model, $\bar{v}_y$ oscillates with a

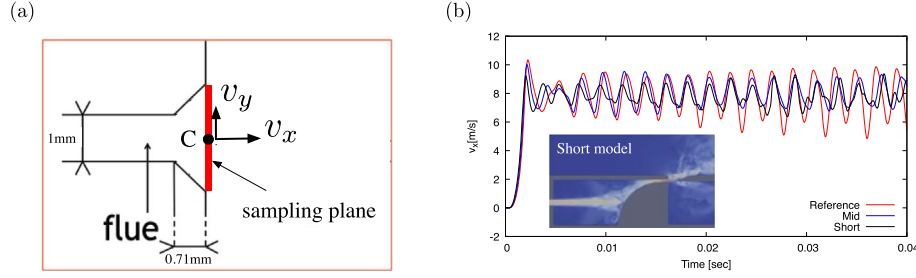


Figure 6. Jet velocity (v_x , v_y) at the flue exit. (a) The observation point C at the center of the flue exit and the sampling plane (the cross-section of the flue opening) over which v_x and v_y are integrated to obtain the averaged values $\bar{v}_x$ and $\bar{v}_y$, respectively. (b) Fluctuations of v_x at the point C for the three models. The inset shows the velocity distribution at $t = 0.04$ s in the operating portion for the Short model.

Table 2. Relative phases among $p_f - p_0$, $p_p - p_0$, $\bar{v}_x$ and $\bar{v}_y$ together with the jet phase $\phi_{x=l}^J \equiv \omega l / V - \theta_0$ at $x = l$ and the normalized displacement $\sin \phi_{x=l}^J$, where ϵ takes small but nonnegligible positive values.

Model	ϕ_{p-f} [rad]	ϕ_{f-vx} [rad]	ϕ_{p-vy} [rad]	ϕ_{vy-vx} [rad]	$\phi_{x=l}^J$ [rad]	$\sin \phi_{x=l}^J$
Reference	2.38 ($\approx 3\pi/4$)	1.54 ($\approx \pi/2$)	0.80 ($\approx \pi/4$)	3.12 ($\approx \pi$)	-0.82 ($\approx -\pi/4$)	-0.73
Mid	1.44 ($\approx \pi/2 - \epsilon$)	1.61 ($\approx \pi/2$)	0.80 ($\approx \pi/4$)	2.25 ($\approx 3\pi/4 - \epsilon$)	0.01 (≈ 0)	0.01
Short	0.55 ($\approx \pi/6$)	1.61 ($\approx \pi/2$)	1.00 ($\approx \pi/3$)	1.16 ($\approx \pi/3 + \epsilon$)	1.07 ($\approx \pi/3$)	0.88

relatively smaller amplitude as a subharmonic oscillation in the steady state and takes positive values in large parts of the time evolution. This indicates that a smaller amount of air is supplied to the pipe giving rise to an acoustic pressure oscillation with a smaller amplitude. In this case, $\bar{v}_x$ lags behind $\bar{v}_y$ by nearly $3\pi/4$ (see Tab. 2). For the Short model, $\bar{v}_y$ stably oscillates taking positive and negative values, although its time average is larger than zero. In this case, $\bar{v}_x$ lags behind $\bar{v}_y$ by about $\pi/3$ (see Tab. 2).

Let us consider subharmonics, which appear in oscillations of $\bar{v}_y$. As shown in Figures 5b, 5d and 5f, the subharmonic of $f = f_a/2$ appears in the spectra of $\bar{v}_y$ for the Reference and Mid models, and almost disappears for the Short model. Such a subharmonic is often observed for nonlinear systems and is caused by the common bifurcation mechanism called the period-doubling bifurcation [32–34]. Subharmonics were observed in oscillations for flue organ pipes and recorders [35–38]. For the Reference and Mid models, the subharmonics of $\bar{v}_y$ should be caused by the period-doubling bifurcation of the vertical oscillation of the jet. Since the pipe is driven by the jet, the subharmonics appear in the acoustic oscillation in the pipe. The subharmonic of $f = 3f_a/2$ that appears in the spectra of $\bar{v}_y$ and p_p for the Reference model and p_p for the Mid model should be caused by the nonlinear interaction between the acoustic oscillation of $f = f_a$ and the subharmonic of $f = f_a/2$ [34]. The influence of the subharmonics on the pressure oscillation in the foot is limited and there are no clear peaks in the spectra of p_f for the Reference and Mid models. Since the x component of the jet velocity v_x is mainly induced by the difference of p_f from the pressure at the mouth opening ($\approx p_0$), a tiny peak of $f = f_a/2$ appears only in the spectrum of $\bar{v}_x$ for the Reference model. According to recent work [35], the irregularity of the jet motion caused by a disturbance rather suppresses the generation of subharmonics.

Thus, a similar mechanism should work for the Short model and a bit irregular jet motion should suppress it. Finally, we consider why sub-harmonics appear for v_x but not for $\bar{v}_x$ for the Reference model. The x component v_x has a velocity profile like a bell shape as a function of y and the profile moves up and down almost keeping its shape owing to the vertical oscillation of the jet involving subharmonics. If v_x is observed at a fixed point, its value changes according to the up-and-down motion of the velocity profile; thus, a subharmonic oscillation should be involved. On the other hand, the influence of subharmonic components disappears for $\bar{v}_x$ due to averaging.

3.4 Jet motion and relative phases among acoustic oscillations in the pipe and foot and the jet velocity

In this subsection, we investigate relative phases among the pressure fluctuation in the pipe $p_p - p_0$, that in the foot $p_f - p_0$ and jet velocity. We also discuss the relative timing of the injection of volume flow to the pipe via the jet oscillation in detail. Table 2 shows the relative phases among $p_p - p_0$, $p_f - p_0$, $\bar{v}_x$, and $\bar{v}_y$. Here, ϕ_{A-B} indicates a phase difference between dynamical variables A and B , namely $\phi_{A-B} = \phi_A - \phi_B$, where ϕ_A and ϕ_B are the phases of oscillations of A and B , respectively. Concretely, ϕ_p , ϕ_f , ϕ_{vx} and ϕ_{vy} denote the phases of $p_p - p_0$, $p_f - p_0$, $\bar{v}_x$ and $\bar{v}_y$, respectively, and ϕ_{p-f} , ϕ_{f-vx} , ϕ_{p-vy} and ϕ_{vx-vy} indicate the phase differences between two corresponding dynamical variables. The phase $\phi_{x=l}^J$ will be defined later.

Concerning ϕ_{p-f} , it takes a value of 2.38 [rad] ($\approx 3\pi/4$) for the Reference model and $p_f - p_0$ and $p_p - p_0$ do not fall in anti-phase synchronization completely, although the system is in the mass-control regime. For the Mid model, $p_f - p_0$ lags behind $p_p - p_0$ by 1.44 [rad], slightly less than $\pi/2$, and the system is in the damping-controlled regime.

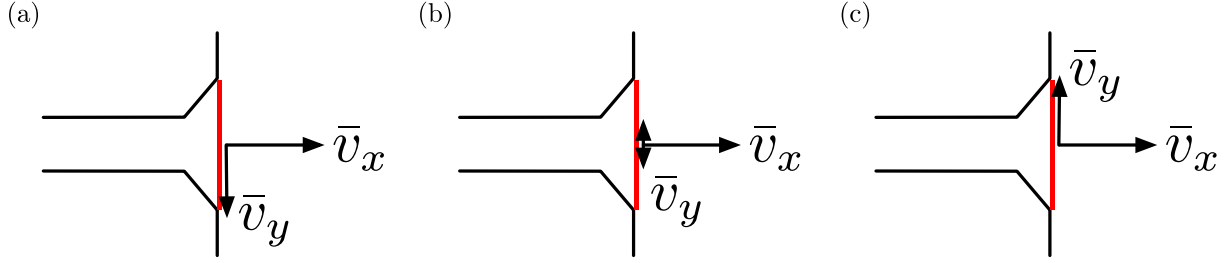


Figure 7. Averaged velocity ($\bar{v}_x, \bar{v}_y$) at the flue exit, when $\bar{v}_x$ takes the local maximum value. (a) Reference model. (b) Mid model. (c) Short model.

For the Short model, ϕ_{p-f} takes a value of 0.55 [rad] ($\approx \pi/6$) and $p_f - p_0$ does not completely synchronize with $p_p - p_0$, although the system is in the stiffness-controlled regime.

Concerning ϕ_{f-vx} , it takes values close to $\pi/2$ for the three models. From the fact that an outgoing acoustic particle velocity at an open end lags behind an acoustic pressure oscillation by $\pi/2$ for a stationary wave in a cavity, the fluctuation of $\bar{v}_x$ is attributed to an acoustic particle velocity u_x induced by an acoustic pressure oscillation, i.e., an acoustic portion of pressure, in the foot. Note that the acoustic particle velocity $\mathbf{u}$ is defined as $\mathbf{u} = \frac{\partial \xi}{\partial t}$, where ξ measures the displacement of the air owing to a sound wave [1].

On the other hand, ϕ_{p-vy} takes values close to $\pi/4$ for the Reference and Mid models and a value close to $\pi/3$ for the Short model. According to the discussion in the previous paragraph, the outgoing acoustic particle velocity u_y at the open end should lag behind the acoustic pressure fluctuation $p_p - p_0$ in the pipe by $\pi/2$. As shown in Appendix (for details, see Ref. [22]), the analysis based on a negative-displacement jet model indicates that the vertical component of jet velocity at the flue exit, namely $\bar{v}_y$, leads u_y by $\pi/4$. Therefore, $\bar{v}_y$ should lag behind $p_p - p_0$ by $\pi/4$ and the values of ϕ_{p-vy} close to $\pi/4$ obtained for the Reference and Mid models are reasonable. For the Short model, the slight difference of ϕ_{p-vy} from $\pi/4$ may be attributed to some additional effects of the jet motion, which are not included in the negative-displacement jet model, e.g., the disturbance of the jet.

Therefore, the values of ϕ_{vy-vx} are obtained from the above phase relations as shown in Table 2. For the Reference model, ϕ_{vy-vx} takes a value of 3.12 [rad] ($\approx \pi$) and $\bar{v}_x$ oscillates in almost anti-phase with $\bar{v}_y$. Then, when $\bar{v}_x$ takes the maximum value in each period, $\bar{v}_y$ takes a large negative value, and a large amount of volume flow is supplied to the pipe at this moment as shown in Figure 7a. Thus, we can expect the jet well to drive the pipe and an acoustic oscillation to be sustained stably. For the Mid model, ϕ_{vy-vx} takes a value of 2.25 [rad], slightly less than $3\pi/4$. Then, a considerable amount of volume flow seems to be injected into the pipe as the Reference model. However, as shown in Figure 5c, $\bar{v}_y$ takes small negative values only in a tiny interval near the locale minimum point in each period. As a result, as shown in Figure 7b, $\bar{v}_y$ takes a small positive or negative value when $\bar{v}_x$ takes the local maximum value; thus, only a small amount of volume flow may be supplied

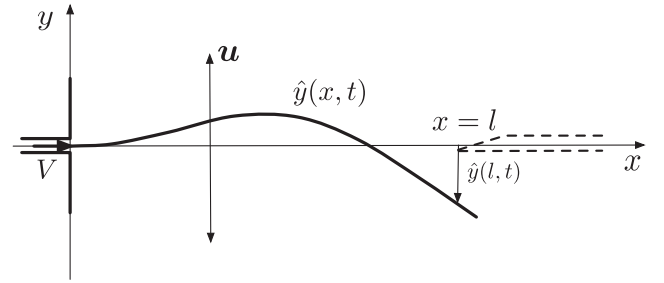


Figure 8. Jet displacement $\hat{y}(x, t)$. $\hat{y}(l, t)$ indicates the displacement at the edge ($x = l$).

to the pipe even if $\bar{v}_y$ takes negative values. This fact should cause the relatively smaller amplitude of $p_p - p_0$. For the Short model, ϕ_{vy-vx} takes a value of 1.16 [rad], slightly larger than $\pi/3$. Thus, as in Figure 7c, $\bar{v}_y$ takes positive values when $\bar{v}_x$ takes the maximum value in each period. Oppositely, $\bar{v}_y$ takes negative values in a certain interval of each period, while $\bar{v}_x$ takes relatively small values. As shown in Figure 5e, a considerable amount of volume flow is injected into the pipe in this interval and should well drive the pipe giving rise to a stable acoustic oscillation.

3.5 Motion of fluid particles on the oscillation jet and relative timing of injecting the volume flow into the pipe

In this subsection, we consider the path of a fluid particle starting from the center of the flue exit, i.e., particle path. Note that the term “fluid particle” means a fluid parcel that is an infinitesimal volume of fluid moving with the fluid flow. Since the previous works based on Howe’s energy corollary (HEC) showed that aerodynamic sound is mainly generated in a downstream area of the jet near the edge [11, 22, 24–26], we expect that the study of how a fluid parcel with the maximum value of $\bar{v}_x$ at the flue exit behaves near the edge gives important knowledge on the generating process of acoustic oscillation in the pipe. For this purpose, we can use the negative-displacement jet model (see Appendix). Indeed, from the previous work [22], the negative-displacement jet model provides a good approximation of the jet motion, e.g., the good estimation of the relative phase between the jet vertical velocity v_y and the outgoing acoustic particle velocity u_y shown in Section 3.4, and can

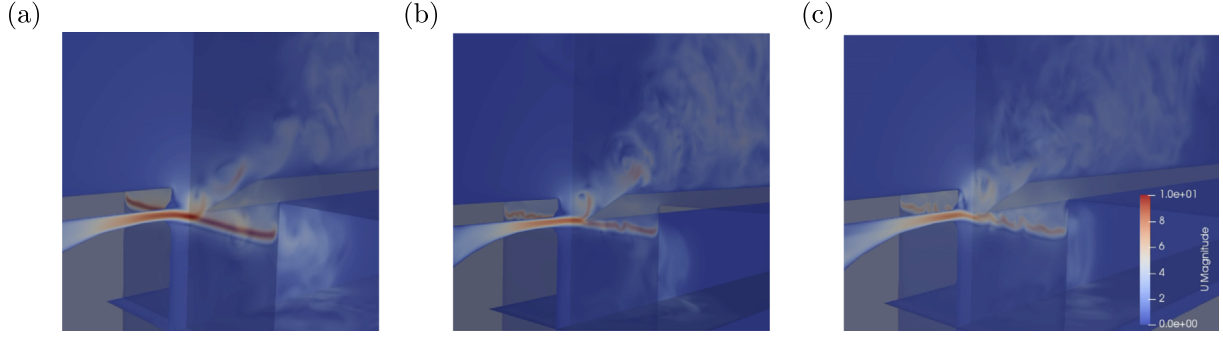


Figure 9. Jet velocity distributions at the lowest positions on two cross-sections: vertical plane along the center line of the pipe and vertical plane along the top of the edge. (a) Reference model at $t = 39.4$ ms. (b) Mid model at $t = 38.2$ ms. (c) Short model at $t = 38.66$ ms.

be used for the study on the acoustic energy generation process based on HEC. Such a simple model allows us to capture an essential feature of jet dynamics. Note that as an alternative method, we may directly trace particle paths with the numerical simulation. However, because of some irregular behavior of the jet near the edge, in particular for the Short model (see Fig. 9), one needs to take a sort of ensemble average of particle paths starting from a parcel with a finite volume near the flue exit, which requires additional post-processing and a careful setting of the ensemble. So, we do not adopt this method in this paper.

First, we estimate the y -coordinate of the fluid particle, when it reaches the vertical line through the center of the edge at $x = l$ as shown in Figure 8. Let us consider the motion of the particle that leaves the center of the flue exit at $t = 0$ at which $\bar{v}_x$ takes the local maximum value. To do this, we assume that the x component of the velocity oscillates as $v_x(x = 0, t) = V + \Delta V_0 \cos \omega t$ at the center of the flue exit, i.e., the origin of the coordinate system. As shown in Figure 6b, V approximately takes a value of 8 m/s and ΔV_0 is significantly smaller than V ; thus, $v_x(0, t)$ can be approximated as $v_x(0, t) \approx V$. We also assume that the time average of the y component of the velocity at the center of the flue exit is approximately zero. However, this is not exactly the case for the Mid model. Then, taking the phase difference ϕ_{vy-vx} into account, the y component of velocity at the center of the flue exit is given as

$$\begin{aligned} \hat{v}_y(x = 0, t) &= v_y(x = 0, t) = v_{y0} \cos(\omega t + \phi_{vy-vx}) \\ &\approx 2\sqrt{2}u_0 \cos(\omega t + \pi/4 + \theta_0), \end{aligned} \quad (1)$$

where $\hat{v}_y(x, t)$ denotes the y component of the velocity of a particle on the center line of the jet, v_{y0} is the amplitude and the last term is given by equation (A3) in Appendix. Namely, the last term is the y component of velocity at the origin predicted by the negative-displacement jet model under the assumption that a uniform acoustic velocity field in the y direction, $u_y = u_0 \cos(\omega t + \theta_0)$ with an initial phase θ_0 and an amplitude u_0 , disturbs the jet. From equation (1), θ_0 is obtained as $\theta_0 = \phi_{vy-vx} - \pi/4$. The particle starting from the center of the flue at $t = 0$ reaches the vertical line through the center of the edge

at $t_l \approx l/V$. From equation (A4), the y -coordinate of the particle at $x = l$ and $t = t_l$ is estimated as

$$\begin{aligned} \hat{y}(l, t_l) &\approx -\frac{u_0 g(\mu l)}{\omega} \sin(\omega(t_l - l/u_w) + \theta_0) \\ &\approx -\frac{u_0 g(\mu l)}{\omega} \sin(-\omega l/V + \phi_{vy-vx} - \pi/4) \\ &\equiv \frac{u_0 g(\mu l)}{\omega} \sin(\phi_{x=l}^J), \end{aligned} \quad (2)$$

where u_w is the phase speed of a hydrodynamic wave of the jet, which is approximated as $u_w = V/2$, and $g(\mu x)$ is an increasing function with a growth rate μ and determines the envelope of the jet oscillation. In the last term, we introduce the jet phase at $t = t_l$ and $x = l$ defined by $\phi_{x=l}^J \equiv \omega l/V - \phi_{vy-vx} + \pi/4 \approx -\omega(t_l - l/u_w) - \phi_{vy-vx} + \pi/4$, and the normalized displacement $\sin \phi_{x=l}^J$, when $\frac{u_0}{\omega} g(\mu l)$ is replaced with one. Table 2 shows $\phi_{x=l}^J$ and $\sin \phi_{x=l}^J$ for the three models.

According to the analysis based on HEC [11, 22, 24–26], acoustic energy is generated twice in every period of the jet motion. Namely, the dominant acoustic energy generation occurs when the jet reaches an area below the edge in a downstroke process and the subdominant one occurs in an area just above the edge in an upstroke process [11]. For the Reference model, $\phi_{x=l}^J$ takes a value of -0.82 [rad] ($\approx -\pi/4$), and $\sin \phi_{x=l}^J$ takes a value of -0.73 ; thus, the dominant condition is satisfied. Indeed, the particle exists on the jet diving under the edge at $t = t_l$, where the acoustic energy is largely generated. This fact shows that the oscillating jet drives the pipe with appropriate timing to sustain the acoustic oscillation, which is demonstrated by the oscillation of $p_p - p_0$ in Figure 5a.

For the Mid model, $\phi_{x=l}^J$ takes a value of 0.01 [rad] and $\sin \phi_{x=l}^J$ takes a value of 0.01 . Thus, when the particle arrives close to the edge, the jet takes an almost horizontal position while going downward. The dominant condition of acoustic energy generation is not satisfied sufficiently; thus, only a little acoustic energy is generated at this moment giving rise to an acoustic oscillation with small amplitude. Since the oscillation of $\bar{v}_y$ is induced by the acoustic oscillation u_y , the oscillating amplitude of $\bar{v}_y$ should become

smaller, while the averaged outgoing flow should be non-negligible. Indeed, $\bar{v}_y$ takes positive values in large parts of the time evolution, and a small amount of volume flow is supplied to the pipe as shown in Figure 5c. In the stationary state, this inappropriate cycle should be repeated, namely falling into a negative cycle. In this sense, the jet drives the pipe with improper timing.

For the Short model, $\phi_{x=l}^J$ takes a value of 1.07 [rad] ($\approx \pi/3$) and $\sin \phi_{x=l}^J$ takes a value of 0.88. Thus, the particle reaches the upper part of the edge at $t = t_l$ but the jet is going downward. No volume flow is substantially supplied to the pipe at this moment and even the subdominant condition of the acoustic energy generation may not be satisfied. However, a particle that starts from the flue exit in the opposite phase reaches the lower part of the edge. Thus, for each period of the jet motion, there is a short interval in which the dominant condition is satisfied: fluid particles taking relatively small values in v_x at the flue exit reach an area below the edge in a downstroke process. Then, a considerable amount of acoustic energy should arise enough to drive the pipe and an acoustic oscillation is well sustained as shown in Figure 5e. In this sense, the Short model can generate acoustic energy under the dominant condition, while the efficiency of the production of acoustic energy by the jet is less than that of the Reference model. The relatively long attack transient should be attributed to the mismatching between the maximum volume flow injection and the dominant condition of acoustic energy generation.

These theoretical predictions are supported by numerical evidence. Figure 9 shows the jet velocity distributions at the lowest positions for the three models. The jet behaves in different manners for the three models. For the Reference model, the strong jet flow dives under the edge and forms a clear envelope. Thus, the jet should drive the pipe generating aerodynamic sound in the dominant condition. For the Mid model, the jet collides with the edge and a large piece of the jet flow goes over the edge. Then, the remaining piece of the jet flow supplied to the pipe is relatively weak and unstable compared to that of the Reference model. Thus, the jet overshoots the edge in large parts of the time evolution and does not sufficiently satisfy the dominant condition at any moment. This fact indicates that the jet drives the pipe with inappropriate timing. For the Short model, a major part of the jet flow dives under the edge but is weak and significantly unstable. This is because the jet is disturbed by the inlet flow in the channel part of the foot as shown in Figure 6b, and a fluid parcel with almost the minimum values of $\bar{v}_x$ at the flue exit reaches near the edge at this moment. The dominant condition is satisfied when the jet dives under the edge despite being weak and unstable. Thus, the jet can drive the pipe fairly well in the stationary state after a relatively long attack transient. Although it is not shown in the figure, when the jet takes an upper position, the relatively strong flow goes over the edge because a parcel with almost the maximum values of $\bar{v}_x$ at the flue exit reaches near the edge at this moment.

Further study on the relative timing between the jet motion and acoustic energy generation based on HEC

combined with the numerical method of tracing particle paths is postponed for future work.

4 Discussion

In this paper, we numerically studied the 3D flue organ pipe model, focusing on the problem of what role the foot, which acts as a Helmholtz resonator, plays in the sound generation process. First, we showed how the detuning of the acoustic oscillation frequency from the resonance frequency of the foot affects the phase difference between the acoustic oscillation in the pipe and the pressure oscillation in the foot. Indeed, the change in the relative phase between the pipe and the foot can be explained by the theory of forced harmonic oscillators (TFHO) [14, 16]. In this scenario, we assumed that the acoustic oscillation in the pipe drives the foot. Thus, the relationship of the Helmholtz resonance frequency f_H of the foot to the acoustic frequency f_a determines the relative phase between the oscillations in the pipe and foot. We observed the three types of responses of the foot to the driving force: the stiffness-controlled response ($f_a < f_H$) for the Short model, the damping-controlled one ($f_a \approx f_H$) for the Mid model, and the mass-controlled one ($f_a > f_H$) for the Reference model. However, for the resonance condition, $f_a \approx f_H$, the acoustic oscillation in the pipe and that in the foot become smaller in amplitude than those for the other models, which contradicts the prediction of TFHO. For this fact, we gave an intuitive explanation that the enhancement of energy transfer from the pipe to the foot owing to resonance consumes the acoustic energy in the pipe as a sort of Helmholtz silencer. We also tried to examine this fact from the viewpoint of forced coupled damped oscillators. However, these interpretations did not clarify the mechanism of reducing the total acoustic energy in the pipe and foot. Thus, this process should be induced by the nonlinear interaction of the jet motion with the acoustic oscillations in the foot and pipe.

Thus, we considered that relative phases among the pressure oscillation in the pipe, that in the foot and the jet velocity are the key to understanding the overall picture of this problem, and showed that total balance among them plays a crucial role in the sound generation process and affects the properties of acoustic and fluid oscillations. Namely, the change of the Helmholtz resonance frequency of the foot changes the relative phase between the pressure oscillation in the pipe and that in the foot so that the relative phase between the x and y components of the jet velocity at the flue exit changes. In other words, the relative timings of the injection of the maximum velocity from the flue against the vertical oscillations of the jet changes depending on the Helmholtz resonance frequency of the foot. The analysis using the negative-displacement jet model indicated that such a change in the relative timing affects the acoustic energy generation process occurring in the downstream area near the edge. Indeed, the jet drives the pipe with appropriate and quasi-appropriate timings to sustain the acoustic oscillations for the Reference and Short models, respectively. Namely, the dominant condition

of the acoustic energy generation given by the analysis based on Howe's energy corollary is more or less satisfied for these models [11]. In contrast, for the Mid model, the dominant condition of the acoustic energy generation is not satisfied sufficiently. Thus, the jet inefficiently drives the pipe with inappropriate timing for generating acoustic energy, and the feedback from the generated acoustic oscillation with a small amplitude makes the jet oscillation smaller in amplitude. Therefore, a small amount of volume flow is supplied to the pipe. This inappropriate cycle should be repeated in the stationary state. These theoretical predictions were supported by numerical evidence.

In this paper, we treated the model with a closed organ pipe. It is worthwhile discussing whether our results apply to open organ pipes. We expect that the results in the mass-controlled and stiffness-controlled regimes are applicable to open flue pipes owing to the stability of oscillations. However, we should pay close attention to treating open flue pipes in the damping-controlled regime for the following reasons. For closed flue pipes, there is a mean upward flow through the mouth opening owing to the effect of feedback from the pipe resonance and recirculation of the airflow [28]. In contrast, a mean upward flow should be reduced for open flue pipes. As a result, the volume flow is supplied more to an open pipe than a closed pipe; thus, more acoustic energy should be generated. On the other hand, Paál *et al.* reported that the jet oscillation becomes larger in amplitude and more unstable for an open flue pipe [27]. Furthermore, it was also reported that the so-called "auto-direction" effect arises for closed flue pipes and the air-jet oscillation is stabilized [29, 30]. From these facts, the jet and acoustic oscillations may become larger in amplitude but more unstable for open flue pipes in the damping-controlled regime. In any case, it should be desirable for both open and closed pipes to operate in either the mass-controlled or stiffness-controlled regimes. These issues should be clarified in future work.

The result of this paper may stimulate one to develop a dynamical model of coupled oscillators combined with the nonlinear jet motion to capture the basic nature of flue organ pipes and to explore relative phases among the acoustic oscillations and the jet motion. Dynamical models involving the propagation of the hydrodynamic wave of the jet as a delay component have been recently developed for flute-like and recorder-like instruments [38, 39]. Such delay models may provide an important clue for modeling flue organ pipes.

An additional but important finding in this paper is the appearance of subharmonics for the vertical jet velocity $\bar{v}_y$ and acoustic oscillation in the pipe for the Reference and Mid models. A dynamical model with delay as mentioned above may explain the bifurcation mechanism of subharmonics. However, the bifurcation of subharmonics is suppressed for the Short model possibly because of the disturbance in the formation of the jet induced by the flow from the inlet in the foot. This suppression mechanism is essentially the same as that reported by Giordano and Saenger [35]. Solving this mechanism is a problem for future work.

Our numerical results may not be directly applied to the sound generation process of real flue organ pipes. Namely, the pipes are arranged over a wind chest and a set of the pipes selected by a given slider's position are pronounced. Even in this case, the foot should still work as a Helmholtz resonator with two necks, and to understand the whole sound generation process we need to consider the phase relationship in oscillations among the pipe, foot and windchest, perhaps including the other pipes connected to the same windchest. If numerical simulation including the effect of the windchest is possible, we may find important knowledge applicable to the design of flue organ pipes. This is an interesting and important problem for future work. For this purpose, an artificial modeling technique for an enclosed volume of gas outside of an open end may be applicable for modeling the volume effect of a windchest. For example, the plenumPressure in OpenFOAM provides, as a pressure boundary condition, a plenum pressure inlet condition using a zero-dimensional model of an enclosed volume of gas upstream of the inlet [40].

In comparing our results with the properties of real flue organ pipes, it is necessary to check which regime, the mass-controlled or the stiffness-controlled regime, real flue organ pipes operate in if it is confirmed that their feet act as Helmholtz resonators. This is because it is more suitable for voicing and tuning adjustments if all pipes or at least pipes in each tonal family, e.g., flutes, diapasons (principals), and strings, have the same characteristics in phase responses. In our numerical calculation, the acoustic oscillation frequencies considerably change depending on the geometry of the foot owing to the coupling between the pipe and foot as shown in Table 1. Therefore, it is important to investigate how the acoustic oscillation frequencies of real flue organ pipes change depending on the geometry of their feet in the mass-controlled and stiffness-controlled regimes from the viewpoint of voicing and tuning adjustments and design of flue organ pipes.

Concerning recorders, the oral cavity and vocal track seem to play the same role as the foot and affect the stability and phase change of acoustic oscillations [12, 13]. It can be assumed that a nearly constant flow is supplied from the lungs to the vocal track. Thus, the situation is similar to our numerical model. Indeed, similar synchronization and anti-phase synchronization were observed experimentally and discussed theoretically [13]. Our numerical results and prediction should be checked by experiments on recorders in future work.

Acknowledgments

The present work was supported by JSPS KAKENHI Grant number JP19K03655, and Joint Usage/Research Center for Interdisciplinary Large-Scale Information Infrastructures (JHPCN) and High Performance Computing Infrastructure (HPCI) in Japan (Project IDs: jh220001, jh230002, hp220278).

Conflicts of interest

The authors declare no conflict of interest.

Data availability statements

The research data associated with this article are included in the supplementary material of this article.

Supplemental material

Supplementary material of this article is available at <https://actaacustica.edpsciences.org/10.1051/aacus/2024060/olm>.

Supplemental material for “Numerical study on role of foot of a flue organ pipe: relative phases in oscillations among pipe, foot and jet”.

References

1. N.H. Fletcher, T.D. Rossing: The physics of musical instruments. 2nd edn., Springer-Verlag, New York. 1998.
2. A. Hirschberg: Aero-acoustics of wind instruments, in: A. Hirschberg, J. Kergomard, G. Weinreich (Eds.), Mechanics of musical instruments, Springer-Verlag, Vienna and New York. 1995, pp. 291–369.
3. B. Fabre, J. Gilbert, A. Hirschberg, X. Pelorson: Aeroacoustics of musical instruments. Annual Review of Fluid Mechanics 44 (2011) 1–25.
4. J.W. Coltman: Jet driven mechanisms in edge tones and organ pipes. Journal of the Acoustical Society of America 60 (1976) 725–733.
5. M.P. Verge, B. Fabre, W.E.A. Mahu, A. Hirschberg, R.R. van Hassel, A.P.J. Wijnands, J.J. de Vries, C.J. Hogendoorn: Jet formation and jet velocity fluctuations in a flue organ pipe. Journal of the Acoustical Society of America 95 (1994) 1119–1132.
6. B. Fabre, A. Hirschberg, A.P.J. Wijnands: Vortex shedding in steady oscillation of a flue organ pipe. Acta Acustica united with Acustica 82 (1996) 863–877.
7. S. Yoshikawa: Jet-wave amplification in organ pipes. Journal of the Acoustical Society of America 103 (1998) 2706–2717.
8. S. Yoshikawa: A pictorial analysis of jet and vortex behaviours during attack transients in organ pipe models. Acta Acustica united with Acustica 86 (2000) 623–633.
9. C. Ségoufin, B. Fabre, M.P. Verge, A. Hirschberg, A.P.J. Wijnands: Experimental study of the influence of the mouth geometry on sound production in a recorder-like instrument: windway length and chamfers. Acta Acustica united with Acustica 86 (2000) 649–661.
10. M. Miyamoto, Y. Ito, T. Iwasaki, T. Akamura, K. Takahashi, T. Takami, T. Kobayashi, A. Nishida, M. Aoyagi: Numerical study on acoustic oscillations of 2D and 3D flue organ pipe like instruments with compressible LES. Acta Acustica united with Acustica 99 (2013) 154–171.
11. R. Tabata, R. Matsuda, T. Koiwaya, S. Iwagami, H. Midorikawa, T. Kobayashi, K. Takahashi: Three-dimensional numerical analysis of acoustic energy absorption and generation in an air-jet instrument based on Howes energy corollary. Journal of the Acoustical Society of America 149 (2021) 4000–4012.
12. J.-M. Chen, D. Laurin, J. Smith, J. Wolfe: Vocal track interactions in recorder performance, in: Proceedings of the 19th International Congress on Acoustics, Madrid, Spain. 2007, pp. 5565–5572.
13. R. Auvray, A. Ernault, S. Terrien, P.Y. Lagrée, B. Fabre, C. Vergez: Effect of changing the vocal tract shape on the sound production of the recorder: an experimental and theoretical study. Acta Acustica united with Acustica 101 (2015) 317–330.
14. S. Tateishi, S. Iwagami, G. Tsutsumi, T. Kobayashi, T. Takami, K. Takahashi: Role of the foot chamber in the sounding mechanism of a flue organ pipe. Acoustical Science and Technology 40, 1 (2019) 29–39.
15. K. Takahashi, S. Iwagami, S. Tateishi, G. Tsutsumi, T. Kobayashi, T. Takami: Anti-phase synchronization between the oscillation in the pipe and that in the foot of a flue organ pipe, in: Proceedings of the 23rd International Congress on Acoustics, Aachen, Germany. 2019, pp. 5565–5572.
16. S. Ikoga, T. Onomata, R. Tabata, S. Iwagami, T. Kobayashi, K. Takahashi: Numerical study on a three-dimensional model of a flue organ pipe: relative phase between pipe and foot, and stability, in: Proceedings of Forum Acusticum 2023, Torino, Italy. 2023, pp. 4439–4442.
17. N.H. Fletcher, S. Thwaites: Wave propagation on an acoustically perturbed jet. Acustica 42 (1979) 323–334.
18. S.A. Elder: Self-excited depth-mode resonance for a wall-mounted cavity in turbulent flow. Journal of the Acoustical Society of America 64 (1978) 877–890.
19. S.A. Elder: Forced oscillations of a separated shear layer with application to cavity flow-tone effects. Journal of the Acoustical Society of America 67 (1980) 774–781.
20. A.W. Nolle: Sinuous instability of a planar air jet: Propagation parameters and acoustic excitation. Journal of the Acoustical Society of America 103 (1998) 3690–3705.
21. B. Fabre, A. Hirschberg: Physical Modeling of Flue Instruments: A Review of Lumped Models. Acta Acustica united with Acustica 86 (2000) 599–610.
22. K. Takahashi, S. Iwagami, T. Kobayashi, T. Takami: Theoretical estimation of the acoustic energy generation and absorption caused by jet oscillation. Journal of the Physical Society of Japan 85 (2014) 044402.
23. M.S. Howe: On the absorption of sound by turbulence and other hydrodynamic flows. IMA Journal of Applied Mathematics 32 (1984) 187–209.
24. A. Bamberger: Vortex sound in flutes using flow determination with endo-piv, in: Forum Acusticum Budapest 2005, 4th European Congress on Acoustics (2005) 665–670.
25. S. Yoshikawa, H. Tashiro, Y. Sakamoto: Experimental examination of vortex-sound generation in an organ pipe: A proposal of jet vortex-layer formation model. Journal of Sound and Vibration 331 (2012) 2558–2577.
26. T. Kobayashi, T. Akamura, Y. Nagao, T. Iwasaki, K. Nakano, K. Takahashi, M. Aoyagi: Interaction between compressible fluid and sound in a flue instrument. Fluid Dynamics Research 46 (2014) 061411.
27. G. Paál, J. Angster, W. Garen, A. Miklós: A combined LDA and flow-visualization study on flue organ pipes. Experiments in Fluids 40 (2006) 825–835.
28. B. Fabre: Flute-like instruments, in: A. Chaigne, J. Kergomard (Eds.), Acoustics of Musical Instruments, Springer-Verlag, New York. 2016, pp. 559–606.
29. N.H. Fletcher: Stopped-pipe wind instruments: Acoustics of the panpipes. Journal of the Acoustical Society of America 117 (2005) 370–374.
30. R. Auvray, B. Fabre, F. Meneses, P. de la Cuadra, P.-Y. Lagrée: Specific features of a stopped pipe blown by a turbulent jet: Aeroacoustics of the panpipes. Journal of the Acoustical Society of America 139 (2016) 3214–3225.
31. T.J. Poinso, S.K. Lele: Boundary conditions for direct simulations of compressible viscous flows. Journal of Computational Physics 101 (1992) 104–129.
32. R.M. May: Simple mathematical models with very complicated dynamics. Nature 261 (1976) 459–467.

33. M.J. Feigenbaum: Quantitative universality for a class of nonlinear transformations. *Journal of Statistical Physics* 19 (1978) 25–52.
34. P. Bergé, Y. Pomeau, Ch Vidal: *L'Ordre dans le Chaos*. Hermann, Paris. 1984.
35. N. Giordano, K.L. Saenger: Study of “half-integer” harmonics in recorder tones and some speculations about their origin. *Journal of the Acoustical Society of America* 154 (2023) 2917–2927.
36. M.-P. Verge: *Aeroacoustics of confined jets, with applications to the physical modeling of recorder-like instruments*. Ph.D. thesis. Technische Universiteit Eindhoven, Eindhoven, Netherlands. 1995.
37. D. Ferrand, C. Vergez, B. Fabre, F. Blanc: High-precision regulation of a pressure controlled artificial mouth: The case of recorder-like instruments. *Acta Acustica united with Acustica* 96 (2010) 701–712.
38. S. Terrien, C. Vergez, B. Fabre: Flute-like musical instruments: A toy model investigated through numerical continuation. *Journal of Sound and Vibration* 332 (2013) 3833–3848.
39. P.T. Nagy, P. Rucz, A. Szabó: Examination of jet growth and jet-drive in the recorder by means of linearized numerical and lumped models. *Journal of Sound and Vibration* 527 (2022) 116857.
40. plenumPressure in OpenFOAM v2306: <https://doc.openfoam.com/2306/tools/processing/boundary-conditions/rtn/derived/inlet/plenumPressure/> (2023).

Appendix

A1 Jet model

According to the textbook written by Fletcher and Rossing [1] and recent studies [17–22], the motion of the center-line of the jet is approximated by a negative-displacement jet model. As shown in Figure 8, when the jet is driven by a uniform acoustic field in the y direction with the velocity $u_y = u_0 \cos(\omega t + \theta_0)$ with an initial phase θ_0 and an amplitude u_0 , the displacement in the y -direction of the center-line of the jet is given by

$$\hat{y}(x, t) = \frac{u_0}{\omega} (\sin(\omega t + \theta_0) - g(\mu x) \sin(\omega(t - x/u_w) + \theta_0)), \quad (\text{A1})$$

where the origin of the coordinate system is taken at the center of the flue exit, u_w is the phase speed of a hydrodynamic wave formed by the jet in the semi-infinite space $x > 0$, and $g(\mu x)$ is an increasing function with a growth rate μ , which is typically taken as an exponential function $g(\mu x) = \exp \mu x$ or a polynomial approximation to it. If the jet velocity V is given at the flue exit, the phase speed u_w is approximately obtained as $u_w \approx V/2$, and the approximation $\mu \approx k = \omega/u_w$ is also used when the jet is not extremely narrow [1, 22].

Ignoring the fluctuation of the x component of flow velocity, we assume that the jet velocity at the center of the flue exit at $(x = 0, y = 0)$ takes a constant value of V . When a fluid particle starts from the point $(x = 0, y = 0)$ at $t = 0$, its x -coordinate is approximated as $x \approx Vt$. Substituting $x \approx Vt$ into equation (A1) and taking the total differential of $\hat{y}(x, t)$ with respect to t , we obtain the y component of the fluid particle velocity on the center line [22]:

$$\begin{aligned} \hat{v}_y(x, t) &= \frac{\partial}{\partial t} \hat{y}(x, t) + v_x \frac{\partial}{\partial x} \hat{y}(x, t) \\ &= u_0 (\cos(\omega t + \theta_0) + (v_x/u_w - 1) g(\mu x) \cos(\omega(t - x/u_w) + \theta_0) \\ &\quad - \frac{\mu v_x}{\omega} g'(\mu x) \sin(\omega(t - x/u_w) + \theta_0)) \end{aligned} \quad (\text{A2})$$

where $g'(x)$ denotes the derivative of $g(x)$. At $x = 0$, equation (A2) gives the y component of the fluid velocity at the flue exit,

$$\begin{aligned} \hat{v}_y(x = 0, t) &= u_0 \left\{ \left[1 + \left(\frac{v_x}{u_w} - 1 \right) g(0) \right] \cos(\omega t + \theta_0) \right. \\ &\quad \left. - \frac{\mu v_x}{\omega} g'(0) \sin(\omega t + \theta_0) \right\} \\ &\approx 2\sqrt{2} u_0 \cos(\omega t + \pi/4 + \theta_0), \end{aligned} \quad (\text{A3})$$

where we make use of $v_x \approx V$, $u_w \approx V/2$ and $\mu \approx k = \omega/u_w$, and also use $g(0) = 1$ and $g'(0) = 1$ in the last line, which are available when $g(\mu x)$ is an exponential function or a polynomial approximation to it. It is numerically confirmed that $\hat{v}_y$ is synchronized with u_y not at the flue exit but at a small distance from the flue exit [11, 22].

Combining equation (A1) with the approximation $x \approx Vt$ provides the trace of the fluid particle $(x(t), \hat{y}(x(t), t))$ that starts from the center of the flue exit at $t = 0$. Since the particle reaches a position $x = l$ at $t_l = l/V$, the y component, $\hat{y}(x(t_l), t_l)$, is approximately obtained as

$$\begin{aligned} \hat{y}(x = l, t_l) &= \frac{u_0}{\omega} (\sin(\omega l/V + \theta_0) - g(\mu l) \\ &\quad \times \sin(\omega(l/V - l/u_w) + \theta_0)) \\ &\approx \frac{u_0}{\omega} (\sin(\omega l/V + \theta_0) - g(\mu l) \sin(-\omega l/V + \theta_0)) \\ &\approx -\frac{u_0}{\omega} g(\mu l) \sin(-\omega l/V + \theta_0), \end{aligned} \quad (\text{A4})$$

where we assumed that $g(\mu l) \gg 1$ in the last line.