DATA and REGRESSION PLOTS

Supplementary to

*Cylinder aeroacoustics: experimental study of*  
*the influence of cross-section shape on spanwise coherence length*

F. Margnat  
W. J. Gonçalves, S. Pinto, C. Noûs

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Abstract

Supplementary charts and tables are provided to the main paper file. The spectra of integral coherence length are given for the other velocities and measurement points than 40 m/s and $P_4$. The data at peak frequencies are tabulated for all of the cases and harmonics, in terms of peak Strouhal number of the integral coherence length, uncertainty range on that Strouhal number, value of that peak, and coherence length based on regression with a decay model, adjusted determination coefficient, maximum lag included and corresponding Strouhal number. A regression plot is then provided for each peak, showing the data and the best Gaussian and Laplacian models.

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2 Database of peak frequencies and coherence length values 4
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Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR</td>
<td>Sectional aspect ratio ($b/d$)</td>
</tr>
<tr>
<td>$b$</td>
<td>Sectional breadth</td>
</tr>
<tr>
<td>$d$</td>
<td>Cylinder diameter, sectional height</td>
</tr>
<tr>
<td>$L$, [$L$]</td>
<td>Spanwise coherence length, [normalized by $d$]</td>
</tr>
<tr>
<td>$L_G$, [$L_G$]</td>
<td>Coherence length for Gaussian decay</td>
</tr>
<tr>
<td>$L_L$, [$L_L$]</td>
<td>Coherence length for Laplacian decay</td>
</tr>
<tr>
<td>$P_1$-$P_4$</td>
<td>Probe locations in $(x, y)$ plane</td>
</tr>
<tr>
<td>$R^2_a$</td>
<td>Adjusted determination coefficient</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number, based on $U_\infty$ and $d$</td>
</tr>
<tr>
<td>$S_{tp}$</td>
<td>Peak Strouhal number in $\Lambda_I$ spectrum</td>
</tr>
<tr>
<td>$S_{tm}$</td>
<td>Strouhal number of modelled coherence decay</td>
</tr>
<tr>
<td>$U_\infty$</td>
<td>Upstream velocity</td>
</tr>
<tr>
<td>$\eta_{max}$</td>
<td>Maximum lag included in the regression</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Coherence function (normalized cross spectrum)</td>
</tr>
<tr>
<td>$\Gamma_0$</td>
<td>Threshold for definition of $\eta_{lim}$</td>
</tr>
<tr>
<td>$\Gamma_p$, [$\Gamma_p$]</td>
<td>Pressure, [velocity] based coherence data</td>
</tr>
<tr>
<td>$\Lambda_I$</td>
<td>Normalized, integral coherence length</td>
</tr>
</tbody>
</table>
1 Integral coherence length spectra

Supplement to Figure 7 in the paper is given here in Fig. 1, which includes the other velocities and measurement points.

Figure 1: Frequency dependant, normalized spanwise coherence length in the flow over cylinders. Influence of velocity, and probe location for rectangular cylinders. Full lines: estimation from spanwise integration of coherence data (equation (3) in the paper); dashed and dash-dotted lines show isocontours of $\Gamma_u = \exp(-\pi/4)$ and $\Gamma_u = \exp(-1)$, respectively, in the $(\eta,St)$ field, corresponding to a coherence decay assumed as either Gaussian or Laplacian, respectively.

$U_\infty = 20 \text{ m/s}$, $P_2$

$U_\infty = 20 \text{ m/s}$, $P_4$

$U_\infty = 40 \text{ m/s}$, $P_2$
Supplement to Figure 8 in the paper is given here in Fig. 2, showing the other measurement points for the square cylinder.

Figure 2: Frequency dependant, normalized spanwise coherence length in the flow over cylinders. Influence of velocity, and probe location for the square section cylinder. Estimation from spanwise integration of coherence data (equation (3) in the paper).
2 Database of peak frequencies and coherence length values

For the 25 flow cases investigated in the present study (shape, inflow velocity, probe location), 75 peaks where detected in the coherence length spectra shown e. g. in Figure 1. Frequency and value at peak are documented in this section in Tables 1-5, as well as parameters of the fitting model (Gaussian or Laplacian decay) if any. When no peak is detected in $\Lambda_I$ spectrum for a given case or harmonic, the case is left blank. A dash indicates when neither the Gaussian nor the Laplacian decays match the data. For the Strouhal number range, when only one St bin returns a maximum in the coherence spectrum at a given lag, it is tabulated instead of the interval, or a dash means that it is equal to St$_p$. For Strouhal numbers, the precision is fixed to that of the signal processing, that is, the tabulated values may be considered as $\pm 0.0005$. For coherence lengths, the values above 1 are rounded to the closest tenth of $d$, consistently with the finest measurement lag resolution.

The ideal peak someone may expect would be when the determination coefficient $R^2_a$ is relatively close to 1, when the integral length $\Lambda_I$ is 10-20% smaller than the regression induced one (see Section 3.2.1 in the paper), when the Strouhal number of a harmonic is close to a multiple of the main peak, when the frequency of maximum integral length (St$_p$) is close to that where the model is found the most faithful to the data (St$_m$) and the Strouhal number range is narrow within which a coherence peak is detected in the spectra at given lag. Distance from these fancies of the mind occurs for several reasons: the model may not be physically relevant; measurement and processing lead to uncertainties in the data affecting the regression quality, which also depends on available lags; excess of coherence biases the estimation of $\Lambda_I$ as well as Strouhal number range; etc.

Users of Figure 11 in the paper should refer to those tables to know the Strouhal number of the peaks and which type of coherence length it contains (either integral or modeled). For engineering applications using those tables, it is recommended to consider value ranges, for frequencies as well as for coherence lengths, in order to obtain the corresponding range for the output (namely, tonal noise level), and decide consequently.

Table 1: Coherence lengths and peak Strouhal numbers for flows over a circular cylinder. See the definition of the tabulated quantities in the nomenclature, estimation methods in Section 3 of the paper and instruction for use hereabove.

<table>
<thead>
<tr>
<th>Re</th>
<th>10,000</th>
<th>13,000</th>
<th>17,000</th>
<th>20,000</th>
<th>27,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st peak</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>St$_p$</td>
<td>0.197</td>
<td>0.194</td>
<td>0.194</td>
<td>0.192</td>
<td>0.192</td>
</tr>
<tr>
<td>St$_m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda_I$</td>
<td>7.8</td>
<td>4.8</td>
<td>4.9</td>
<td>4.8</td>
<td>4.5</td>
</tr>
<tr>
<td>$\Lambda_G$ (R$^2_a$, $\eta_{max}$)</td>
<td>5.0 (0.96, 3.4)</td>
<td>4.8 (0.98, 4.1)</td>
<td>4.6 (0.99, 4.3)</td>
<td>4.4 (0.99, 4.5)</td>
<td>4.4 (0.99, 4.4)</td>
</tr>
<tr>
<td>$\Lambda_L$ (R$^2_a$, $\eta_{max}$)</td>
<td>0.201</td>
<td>0.198</td>
<td>0.198</td>
<td>0.195</td>
<td>0.193</td>
</tr>
<tr>
<td>2nd peak</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>St$_p$</td>
<td>0.412</td>
<td>0.396</td>
<td>0.389</td>
<td>0.383</td>
<td>0.380</td>
</tr>
<tr>
<td>St$_m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda_I$</td>
<td>2.5</td>
<td>1.8</td>
<td>1.7</td>
<td>1.6</td>
<td>1.5</td>
</tr>
<tr>
<td>$\Lambda_L$ (R$^2_a$, $\eta_{max}$)</td>
<td>2.8 (0.91, 5.0)</td>
<td>-</td>
<td>2.0 (0.95, 3.5)</td>
<td>1.9 (0.92, 2.8)</td>
<td>1.8 (0.93, 2.6)</td>
</tr>
<tr>
<td>3rd peak</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>St$_p$</td>
<td>0.584</td>
<td>0.577</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>St$_m$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda_I$</td>
<td>0.75</td>
<td>0.59</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda_L$ (R$^2_a$, $\eta_{max}$)</td>
<td>0.92 (0.76, 1.8)</td>
<td>0.67 (0.89, 1.3)</td>
<td>0.584</td>
<td>0.577</td>
<td></td>
</tr>
</tbody>
</table>
Table 2: Coherence lengths and peak Strouhal number for flows over a square cylinder (AR = 1), first peak (lift peak).

<table>
<thead>
<tr>
<th>Re</th>
<th>$St_p$</th>
<th>$\Lambda_I$</th>
<th>$\Lambda_G (R_{D}^2, \eta_{\text{max}})$</th>
<th>$St_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$P_3$</td>
<td>$P_4$</td>
</tr>
<tr>
<td>6,700</td>
<td>0.127</td>
<td>0.129</td>
<td>0.127</td>
<td>0.132</td>
</tr>
<tr>
<td></td>
<td>(0.123-0.127)</td>
<td>(0.126-0.132)</td>
<td>(0.126-0.129)</td>
<td>(0.127-0.132)</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>5.2</td>
<td>3.7</td>
<td>3.0</td>
</tr>
<tr>
<td></td>
<td>4.2 (0.97, 3.8)</td>
<td>3.4 (0.97, 3.3)</td>
<td>3.2 (0.96, 2.9)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.126</td>
<td>0.128</td>
<td>0.126</td>
<td>0.128</td>
</tr>
</tbody>
</table>

Table 3: For the square cylinder (AR = 1), second peak (drag peak).

<table>
<thead>
<tr>
<th>Re</th>
<th>$St_p$</th>
<th>$\Lambda_I$</th>
<th>$\Lambda_G (R_{D}^2, \eta_{\text{max}})$</th>
<th>$St_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1$</td>
<td>$P_2$</td>
<td>$P_3$</td>
<td>$P_4$</td>
</tr>
<tr>
<td>6,700</td>
<td>0.252</td>
<td>0.255</td>
<td>0.255</td>
<td>0.258</td>
</tr>
<tr>
<td></td>
<td>(0.248-0.266)</td>
<td>(0.253-0.258)</td>
<td>(0.249-0.260)</td>
<td>(0.255-0.262)</td>
</tr>
<tr>
<td></td>
<td>2.1</td>
<td>2.5</td>
<td>1.7</td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td>2.7 (0.91, 2.4)</td>
<td>1.8 (0.99, 2.6)</td>
<td>1.9 (0.93, 3.0)</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>0.256</td>
<td>0.257</td>
<td>0.257</td>
<td>0.261</td>
</tr>
</tbody>
</table>

| 13,000 | 0.248 | 0.251 | 0.250 | 0.251 |
|        | (0.247-0.253) | (0.220-0.255) | (0.248-0.253) | (0.249-0.254) |
|        | 2.2 | 2.0 | 2.3 | 2.5 |
|        | - | - | 2.6 (0.94, 5.0) | - |
|        | - | - | 0.250 | - |

| 27,000 | 0.249 | 0.256 | 0.250 | 0.250 |
|        | (0.249-0.252) | (0.225-0.257) | (0.249-0.252) | (0.249-0.252) |
|        | 2.3 | 1.5 | 3.1 | 3.2 |
|        | - | - | 5.7 (0.97, 6.0) | - |
|        | - | - | 0.250 | - |

† The data follows a Laplacian decay here, that is $\Lambda_L = 2.7$. 

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Table 4: For the square cylinder (AR = 1), third peak.

<table>
<thead>
<tr>
<th>Re</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,700</td>
<td>0.376</td>
<td>0.381</td>
<td>0.392</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.377-0.395)</td>
<td>(0.381-0.390)</td>
<td>(0.386-0.397)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>1.0</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_G (R_{tr}^2, \eta_{max})$</td>
<td>-</td>
<td>1.1 (0.93, 1.9)</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>$\text{St}_m$</td>
<td>-</td>
<td>0.381</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>13,000</td>
<td>0.376</td>
<td>0.374</td>
<td>0.376</td>
<td>0.378</td>
</tr>
<tr>
<td></td>
<td>(0.371)</td>
<td>(0.373-0.381)</td>
<td>(0.374-0.381)</td>
<td>(0.374-0.382)</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>1.6</td>
<td>2.0</td>
<td>1.9</td>
</tr>
<tr>
<td>$\Lambda_L (R_{tr}^2, \eta_{max})$</td>
<td>0.8 (0.53, 1.6)</td>
<td>1.7 (0.92, 3.5)</td>
<td>2.3 (0.80, 5.0)</td>
<td>2.0 (0.82, 5.0)</td>
</tr>
<tr>
<td>$\text{St}_m$</td>
<td>0.376</td>
<td>0.375</td>
<td>0.376</td>
<td>0.378</td>
</tr>
<tr>
<td>27,000</td>
<td>0.374</td>
<td>-</td>
<td>0.375</td>
<td>0.377</td>
</tr>
<tr>
<td></td>
<td>(0.384-0.389)</td>
<td>(0.374-0.378)</td>
<td>(0.376-0.380)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>2.3</td>
<td>2.4</td>
<td>2.0</td>
</tr>
<tr>
<td>$\Lambda_L (R_{tr}^2, \eta_{max})$</td>
<td>0.7 (0.83, 1.3)</td>
<td>-</td>
<td>2.8 (0.97, 5.0)</td>
<td>2.5 (0.95, 4.0)</td>
</tr>
<tr>
<td>$\text{St}_m$</td>
<td>0.374</td>
<td>-</td>
<td>0.375</td>
<td>0.377</td>
</tr>
</tbody>
</table>

Table 5: For the square cylinder (AR = 1), fourth peak.

<table>
<thead>
<tr>
<th>Re</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6,700</td>
<td>0.503</td>
<td>0.508</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.509)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Lambda_L (R_{tr}^2, \eta_{max})$</td>
<td>0.8 (0.80, 1.3)</td>
<td>0.8 (0.73, 1.6)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\text{St}_m$</td>
<td>0.504</td>
<td>0.508</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13,000</td>
<td>0.500</td>
<td>0.501</td>
<td>0.504</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.502)</td>
<td>(0.504)</td>
<td>(0.504-0.509)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.7</td>
<td>0.6</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_L (R_{tr}^2, \eta_{max})$</td>
<td>-</td>
<td>-</td>
<td>0.9 (0.78, 2.0)</td>
<td></td>
</tr>
<tr>
<td>$\text{St}_m$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.504</td>
</tr>
<tr>
<td>27,000</td>
<td>0.512</td>
<td>0.499</td>
<td>0.502</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.512-0.541)</td>
<td>(0.498-0.502)</td>
<td>(0.505)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>0.9</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>$\Lambda_L (R_{tr}^2, \eta_{max})$</td>
<td>-</td>
<td>-</td>
<td>0.9 (0.86, 1.7)</td>
<td></td>
</tr>
<tr>
<td>$\text{St}_m$</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.502</td>
</tr>
</tbody>
</table>
Table 6: Coherence lengths and peak Strouhal number for flows over a AR = 2 rectangular cylinder.

<table>
<thead>
<tr>
<th>Re</th>
<th>13,000</th>
<th>27,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_2$</td>
<td>$P_4$</td>
</tr>
</tbody>
</table>

1st peak

- $St_p$
  - 0.081
  - (0.075-0.083)
  - 0.081
  - (0.076-0.082)
  - 0.078
  - (0.074-0.079)
  - 0.079
  - (0.075-0.078)

- $\Lambda_I$
  - 19.3
  - 14.9
  - 17.3
  - 13.5

2nd peak

- $St_p$
  - 0.142
  - (0.141-0.143)
  - 0.141
  - (0.140-0.142)
  - 0.145
  - (0.144-0.146)
  - 0.146
  - (0.143-0.147)

- $\Lambda_I$
  - 11.7
  - 11.1
  - 9.3
  - 7.4

3rd peak

- $St_p$
  - 0.220
  - (0.217-0.222)
  - 0.223
  - (0.221-0.226)

- $\Lambda_I$
  - 3.24
  - 2.41

- $\Lambda_L (R_{ar}^+, \eta_{max})$
  - 3.3 (0.80, 7.0)
  - 2.8 (0.85, 6.0)

- $St_m$
  - 0.219
  - 0.223

4th peak

- $St_p$
  - 0.282
  - (0.274-0.285)

- $\Lambda_I$
  - 2.0

Table 7: For the AR = 3 rectangular cylinder.

<table>
<thead>
<tr>
<th>Re</th>
<th>13,000</th>
<th>27,000</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_2$</td>
<td>$P_4$</td>
</tr>
</tbody>
</table>

1st peak

- $St_p$
  - 0.164
  - (0.162-0.165)
  - 0.164
  - (0.162-0.164)
  - 0.161
  - (0.159-0.161)
  - 0.160
  - (0.159-0.161)

- $\Lambda_I$
  - 18.5
  - 17.1
  - 21.5
  - 20.4

- $\Lambda_G (R_{ar}^+, \eta_{max})$
  - 14.1 (0.80, 9.3)
  - 15.9 (0.96, 17.1)
  - 23.0 (0.96, 21.5)
  - 21.4 (0.98, 20.4)

- $St_m$
  - 0.164
  - 0.164
  - 0.161
  - 0.161

2nd peak

- $St_p$
  - 0.322
  - (0.320-0.323)
  - 0.323
  - (0.321-0.323)
  - 0.319
  - (0.317-0.319)
  - 0.319
  - (0.317-0.319)

- $\Lambda_I$
  - 2.5
  - 6.6
  - 6.8
  - 9.1

- $\Lambda_L (R_{ar}^+, \eta_{max})$
  - -
  - 8.1 (0.91, 14.0)
  - 8.2 (0.84, 13.0)
  - 11.2 (0.94, 17.0)

- $St_m$
  - -
  - 0.323
  - 0.319
  - 0.319

3rd peak

- $St_p$
  - 0.483
  - (0.480-0.486)

- $\Lambda_I$
  - 0.99

- $\Lambda_L (R_{ar}^+, \eta_{max})$
  - 1.1 (0.66, 2.4)

- $St_m$
  - 0.483
3 Plots of coherence data and decay models at peak frequencies

For all of the 75 peaks, the data is shown at $St_m$ (see nomenclature, and more details in the paper). When no model is tabulated in the paper, the $St_m$ of the present document has been selected as the best compromise between high determination coefficient and high value of model coherence length. In the corresponding plots here, one can see how unsatisfying is the model anyway, justifying the tabulation of the integral length ($A_{\ell}$) only. For instance, in Fig. 7 top, the Gaussian model seems fair ($R^2 = 0.90$) but it needs to include lags up to $L_{\ell}/2$ only, and then $L_G$ is very smaller than $L_{\ell}$. The Laplacian model yields $L_L$ close to $L_{\ell}$ but does not fit the data at short lags. Other guidelines for this documents are in Fig. 3.

\[
S_{\ell} = 0.126 + 3*0.001 = 0.129
\]

The value at $S_{\ell}$ is 

\[
3.82 + 1.41 = 5.22
\]

$St = 0.126(+3)$, $L_G = 3.82(+1.41)d$

$L_G = 4.23d$

$L_L = 5.23d$

\[R^2(L_G) = 0.97\]

\[R^2(L_L) = 0.72\]

Figure 3: How to read the following figures. This case is herebefore in Tab. 2 ($P_2$, Re = 6,700).
Figure 4: Circular, 1st peak.
Figure 5: Circular, 2nd peak.
Figure 6: AR = 1, 10 m/s, 1st peak.

- $St = 0.125(+2), \quad L_I = 2.93(+0.07)d$
- $St = 0.126(+3), \quad L_I = 3.82(+1.41)d$
- $St = 0.128(-1), \quad L_I = 3.28(+0.43)d$
- $St = 0.128(+4), \quad L_I = 2.95(+0.10)d$

- $L_G = 1.33d$
- $L_G = 4.23d$
- $L_G = 3.42d$
- $L_G = 3.17d$

- $L_L = 3.35d$
- $L_L = 5.21d$
- $L_L = 4.71d$
- $L_L = 4.01d$
Figure 7: AR = 1, 10 m/s, 2nd peak.
Figure 8: AR = 1, 10 m/s, 3rd peak.
Figure 9: AR = 1, 10 m/s, 4th peak.
Figure 10: AR = 1, 20 m/s, 1st peak.
Figure 11: AR = 1, 20 m/s, 2nd peak.
Figure 12: AR = 1, 20 m/s, 3rd peak.
Figure 13: AR = 1, 20 m/s, 4th peak.
Figure 14: AR = 1, 40 m/s, 1st peak.
Figure 15: AR = 1, 40 m/s, 2nd peak.
Figure 16: AR = 1, 40 m/s, 3rd peak.

\[ \text{St} = 0.374(\pm 0), \quad L_I = 0.58(+0.00)d \]

\[ \sqrt{\log(\Gamma)} \]

\[ L_G = 0.74d \]

\[ R^2(G) = 0.76 \]

\[ L_L = 0.67d \]

\[ R^2(L) = 0.83 \]

\[ \text{St} = 0.384(\pm 0), \quad L_I = 2.31(+0.00)d \]

\[ \sqrt{\log(\Gamma)} \]

\[ L_G = 2.56d \]

\[ R^2(G) = 0.12 \]

\[ L_L = 3.04d \]

\[ R^2(L) = 0.71 \]

\[ \text{St} = 0.375(\pm 0), \quad L_I = 2.35(+0.00)d \]

\[ \sqrt{\log(\Gamma)} \]

\[ L_G = 2.54d \]

\[ R^2(G) = 0.33 \]

\[ L_L = 2.79d \]

\[ R^2(L) = 0.97 \]

\[ \text{St} = 0.377(\pm 0), \quad L_I = 2.00(+0.00)d \]

\[ \sqrt{\log(\Gamma)} \]

\[ L_G = 2.18d \]

\[ R^2(G) = 0.50 \]

\[ L_L = 2.50d \]

\[ R^2(L) = 0.95 \]
Figure 17: AR = 1, 40 m/s, 4th peak.
Figure 18: AR = 2, 1st peak.

St = 0.081(±0.0), \( L_1 = 19.28(±0.00)d \)

\( \Gamma \)

\( \eta \)

\( \sqrt{\log \Gamma} \)

\( -\log \Gamma \)

\( R^2(L_G) = 0.83 \)

\( R^2(L_L) = 0.88 \)

\( L_G = 10.09d \)

\( L_L = 34.96d \)

St = 0.081(±0.0), \( L_1 = 14.86(±0.00)d \)

\( \Gamma \)

\( \eta \)

\( \sqrt{\log \Gamma} \)

\( -\log \Gamma \)

\( R^2(L_G) = 0.66 \)

\( R^2(L_L) = 0.86 \)

\( L_G = 7.32d \)

\( L_L = 18.96d \)

St = 0.078(±0.0), \( L_1 = 17.26(±0.00)d \)

\( \Gamma \)

\( \eta \)

\( \sqrt{\log \Gamma} \)

\( -\log \Gamma \)

\( R^2(L_G) = 0.98 \)

\( R^2(L_L) = 0.84 \)

\( L_G = 8.81d \)

\( L_L = 27.98d \)

St = 0.079(±0.0), \( L_1 = 13.49(±0.00)d \)

\( \Gamma \)

\( \eta \)

\( \sqrt{\log \Gamma} \)

\( -\log \Gamma \)

\( R^2(L_G) = 0.96 \)

\( R^2(L_L) = 0.74 \)

\( L_G = 7.83d \)

\( L_L = 18.47d \)

Figure 18: AR = 2, 1st peak.
Figure 19: AR = 2, 2nd peak.
Figure 20: AR = 2, 3rd peak.

Figure 21: AR = 2, 4th peak.
Figure 22: AR = 3, 1st peak.
Figure 23: AR = 3, 2nd peak.
Figure 24: AR = 3, 3rd peak.

Figure 25: Circular, 3rd peak.