Parametric model of young infants’ eardrum and ear canal impedances supporting immittance measurement results. Part I: Development of the model

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Abstract – Wideband acoustic immittance (WAI) measurements provide an objective means to detect pathological middle ear conditions. However, for ears of young infants, it is still difficult to make clear statements about the middle ear status based on WAI measurements. In order to gain a better understanding of WAI data obtained in young infants’ ears, a parametric electro-acoustic model of the ear canal and the middle ear of young infants is proposed. In this first part of the two-part paper, the development of the model for the healthy ear is presented. Based on an existing model for adult ears, the presented model is adapted to young infants’ ears, uses parameters suited to represent physiological properties, and uses a smaller number of parameters in order to reduce model complexity. A comparison of the acoustic input impedance of the ear predicted by the model with real ear measurements in young infants’ ears showed a good agreement in the main characteristics. Model predictions show that the medium frequency range (about 1–3 kHz) of the acoustic input impedance of the ear is dominated by the properties of the eardrum and the middle ear, indicating that pathological middle ear conditions can preferably be detected in this frequency range.

Keywords: Acoustic impedance, Admittance, Middle ear, Outer ear, Screening

1 Introduction

Pathological middle ear conditions occur frequently in early infancy. In the first months of life, middle ear effusion, i.e. fluid in the middle ear resulting from otitis media, is the most frequent pathological middle ear condition. In [1] a prevalence of 74% for middle ear effusion was found for children in the first 6 months of life, and in [2] a prevalence of 47.8% was found for infants aged between 2 months and 6 months. Less frequent conditions are e.g. oedematous middle ear, negative pressure difference between middle ear and ear canal, and middle ear dysplasia. Besides treatments that may or may not be necessary as a result, such conditions also have consequences for diagnostic tests like those used within universal newborn hearing screening programs, because the sound has to travel through the middle ear in order to trigger a response. If an otoacoustic emissions test is applied, the middle ear has to be traveled even twice, first by the stimulus and second by the emission from the cochlea back to the ear canal. Such tests will also be affected by the non-pathological occurrence of amniotic fluid in the middle ear, typically present right after birth up to a few days.

Traditionally, tympanometry at a single frequency is used to detect pathological middle ear conditions. In the last years broadband immittance-based methods were investigated for their suitability and potential superiority in middle ear diagnostics. In this context the term immittance comprises the acoustic impedance, the acoustic admittance, as well as further measures derived from these quantities. The measures of these methods are grouped under the term “wideband acoustic immittance (WAI)”.

In 2013 a consensus statement was formulated by several researchers in that field. It was concluded that WAI, as a tool to improve the diagnostics of middle ear disorders is very promising, however, further research is needed especially to increase the database for both, normative data and data for different pathologies [3].

Today, it is still difficult to make clear statements about the middle ear status of young infants based on WAI measurements. One important aspect is that middle ears of young infants exhibit different acousto-mechanical properties in comparison to older children and adults. Furthermore, WAI is based on acoustic measurements in the ear fluid in the middle ear, typically present right after birth up to a few days.

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canal and it is known that, in the first months of life, the acoustic properties of the ear canal also significantly differ from those at an older age.

In order to gain a better understanding of WAI, knowledge about the impact of different factors like compliant ear canal walls, ear canal shape, and the healthy middle ear versus different pathological middle ear conditions on WAI-measures would be valuable. While some early WAI studies with young infants [4–8] report complex impedance/admittance data together with measures derived from this data, in many recent WAI studies with young infants the only immittance measure reported is either the energy reflectance or the energy absorbance [9–13]. Both require an assumption or estimation regarding the value of the tube wave impedance associated with the ear canal area at the probe tip, which is often not consistent over studies and measurement devices, making the results hard to compare. In addition, they neglect phase information which could potentially be useful to identify acousto-mechanical properties of the residual ear canal and its termination by the eardrum and the middle ear.

In order to be able to verify the suitability of WAI as a diagnostic tool for young infants, certainty of the middle ear status and measures which are independent of the particular measurement device being used are needed. It would be helpful to have detailed knowledge of the transfer of sound in the ear canal to the eardrum, and more specifically to the middle ear. Acousto-mechanical transfer models would provide such detailed knowledge. Many models of this kind have been proposed, e.g. [14–21], which, in general, share many properties but differ in details. The more recent models in particular are mostly further developments of the earlier models. However, they have been developed specifically for adults. Since it is known that several potentially relevant properties of the ear canal and the middle ear differ significantly between adults and young infants, they can in general not be used for young infants.

Therefore, in this paper, a parametric electro-acoustic model of the ear canal and the middle ear of young infants, to be used to predict the acoustic input impedance at the eardrum at ambient pressure, will be presented. The model is based on the model for adults according to [18–20] with the following aims of adaptation: (1) differences between infants’ and adults’ ears should be taken into account, (2) model parameters should be suited to represent physiological properties, and (3) model complexity in terms of the number parameters should be reduced. The adult model from [18–20] was chosen as a starting point because it proved useful in previous studies of the authors on the development of individualized models for the prediction of the sound pressure at the eardrum [22–24] and on the development of implantable middle-ear sensors [25]. It should be noted that in this study we use the same concept of the drum impedance as in [18–20, 26], i.e. the impedance seen when exciting the eardrum by a plane wave impinging perpendicularly onto the drum surface, whereas in many other studies, the drum impedance is referred to as the impedance at or shortly behind the entrance to the so-called inclined coupling region, which in turn is composed of the inclined eardrum and the air space below it.

The objective of the present work is that the adapted model can be used to predict the ability of immittance-basedmeasurements in the ear canal to detect certain different pathological middle ear conditions. Hence its focus is the sound transfer in the ear canal and the acoustic input impedance of the eardrum and not the transfer through the middle ear.

In a companion paper, the model will be extended to predict the influence of pathological middle ear conditions in young infants’ ears on the acoustic input impedance of the ear in order to investigate implications for WAI-measurements. A MATLAB-implementation of the complete model can be found in [27].

2 Development of the model

In the following, the proposed parametric electro-acoustic model of infants’ middle ears and ear canals is described. The parameters of this model are linked to physiological properties. The starting point of the development was the model of the ear canal and middle ear for adults according to [18], which was adapted to model infants’ ears. Furthermore, adaptations were made in order to reduce the model complexity by removing unimportant parameters. As will be explained in the following, reasons for parameters to be unimportant are (1) they are not needed for young infants’ ears, (2) they model effects which were not strongly supported by the available measurement data in [18], or (3) their influence on the resulting input impedance of the eardrum is small.

2.1 Existing parametric model of the ear for adults [18]

The transmission-line model proposed in [18] is depicted in Figure 1. It consists of the acoustic two-port $C$ representing the ear canal, the acoustic impedance of the middle ear cavities $Z_{av}$, the acousto-mechanical kernel two-port $K$ representing eardrum, malleus and incus, a mechanical admittance $y_b$, representing the incudostapedial joint, the mechnano-acousical stapes two-port $S$ and the mechanical admittance of the cochlea $y_c$. Additionally the model includes a Thévenin-equivalent acoustic source with the sound source pressure $p_s$ and the source acoustic impedance $Z_s$ connected to the ear canal two-port. As can be seen in Figure 1, a topology-preserving analogy between electric, acoustic and mechanic domain was used, where electrical voltage corresponds to acoustical pressure $p$ and mechanical velocity $v$. The electrical current corresponds to acoustical volume velocity $q$ and mechanical force $F$.

2.2 Adaptation of the parametric model to infants’ ears

2.2.1 Ear canal

In [18] it was proposed to model the ear canal by a stepped circular duct with rigid walls. The sound
transmission of the duct is defined by the width $\Delta_i$ and the cross sectional area $A_i$ of each slice $i$, and is given by

$$\begin{align*}
\left[ \frac{p_{ec}}{q_{ec}} \right] &= \left( \prod_i \begin{bmatrix} \cosh(\gamma_i \Delta_i) & \sinh(\gamma_i \Delta_i) Z_{tw} \\ \sinh(\gamma_i \Delta_i) / Z_{tw} & \cosh(\gamma_i \Delta_i) \end{bmatrix} \right) \\
&\times \begin{bmatrix} 1/Z_t & 0 \\ 1 & 1 \end{bmatrix} \left[ \begin{bmatrix} p_D \\ q_D \end{bmatrix} \right],
\end{align*}$$

\( (1) \)

with $\gamma$ the propagation constant, $Z_{tw}$ the wave impedance, both depending on the cross-sectional area $A$, and with $Z_t$ the impedance of the tapered end of the ear canal between the points D and T, see Figure 2. Both $\gamma_i$ and $Z_{tw}$ were computed according to [28], considering thermal and viscous losses for smooth wall surfaces, resulting in a complex tube wave impedance and a complex propagation constant with a real part $\alpha$ and an imaginary part $\beta$. In [19] it was proposed to increase the damping (i.e. the real part of the propagation constant) according to

$$\gamma = g\alpha + j\beta, \quad (2)$$

with $g = 3$ to account for non-smooth ear canal walls. The value of $g$ was directly taken from [19] where it was estimated from reflectance measurements of rigidly terminated cadaver ear canal slices. The wave impedance is then approximated by

$$Z_{tw} = \frac{\rho c}{A} \gamma, \quad (3)$$

with $\rho$ the mass density of air and $c$ the speed of sound.

This model implies the first azimuthal mode being outside the relevant frequency range, the sound pressure at the umbo producing the most relevant contribution to the force acting on the manubrium, and rigid ear canal walls.

Physiological differences between adults’ and infants’ ear canals considered for our adaptation are the smaller dimensions and the much more compliant ear canal walls [29, 30]. From birth up to an age of 6 months, the typical distance from the tragus to the innermost point at the ear-drum ranges from 21 mm to 26.1 mm [4, 31, 32]. The depth of the concha was reported to be between 7 mm and 8.6 mm in [31], resulting in an ear canal length ranging from 14 mm to 17.5 mm. In comparison, for adults a mean value of 23 mm ear canal length was reported in [4]. The mean ear canal diameter for infants up to an age of 6 months ranges from 4.4 mm to 6.3 mm, see [4]. For adults, a value of 10.4 mm diameter was reported in [4]. It should be noted, that the mentioned values of ear canal geometry from [4, 31, 32] were determined from acoustic measurements with the underlying assumption of a cylindrical ear canal with rigid termination.

The walls of the ear canal are much softer and more flexible in young infants than in adults, see e.g. [33]. In [4] a resonant behavior of ear canal walls was found up to an age of 6 months. In order to consider the softer ear canal walls in the model, we propose to introduce a wall impedance

$$Z_{Wi} = W_{wi} + j\omega M_{wi} + \frac{1}{j\omega N_{wi}}, \quad (4)$$

for every ear canal slice $i$ with the wall resistance $W_w$, the acoustic mass $M_w$ and the wall compliance $N_w$. Both the acoustic mass and the compliance are determined by the properties of the soft tissue in the ear canal, as

$$M_w = \frac{\rho st d_{st}}{A_{st}}, \quad (5)$$

$$N_w = \frac{d_{st}A_{st}}{E_{st}}, \quad (6)$$

Figure 1. General model structure for sound transmission from the ear canal into the cochlea according to [18].

Figure 2. Sketch of the ear canal slice model showing the ear canal entrance E, the definition of the drum area D and the termination point T.
with \( \rho_{st} \) the density, \( d_{st} \) the thickness, \( A_{st} \) the surface area, and \( E_{st} \) the Young’s modulus of the soft tissue. The damping is modeled with a frequency independent damping ratio given by

\[
\zeta_{st} = 0.5 \left( \frac{\alpha}{\omega} + \beta \omega \right). \tag{7}
\]

For the damping constants \( \alpha \) and \( \beta \) it is assumed that \( \alpha + \beta \omega = \beta \omega \). The frequency-dependent wall resistance is then given by

\[
W_w = \frac{\alpha M_w + \beta}{N_w}. \tag{8}
\]

The proposed modeling of the damping is motivated by the work in [34] in which a finite-element model of the ear canal and middle ear of a 22 days old infant was proposed. In [34], three sets of values were given for the ear canal wall parameters \( E_{st}, \rho_{st} \) and \( \zeta_{st} \). The three sets, referred to as low impedance, baseline and high impedance, were chosen to cover a realistic range of values. The values given were \( E_{st}: 20 \, \text{kPa}, 210 \, \text{kPa} \) and \( 400 \, \text{kPa} \), \( \rho_{st}: 1000 \, \text{kg/m}^3 \), \( 1100 \, \text{kg/m}^3 \) and \( 1200 \, \text{kg/m}^3 \), \( \zeta_{st}: 0.1, 0.25 \) and \( 0.4 \). No values were given in [34] for the thickness \( d_{st} \) and the area \( A_{st} \) of the soft tissue in the ear canal. In order to reproduce the modeling results of the ear canal wall impedance \( Z_{w} \) in [34], we adopt the values of \( E_{st} \) and \( \rho_{st} \). For the damping ratio \( \zeta_{st} \) modified values of \( 0.5, 0.6, 0.9 \) are used. The thickness \( d_{st} \) is assumed to be \( 3 \, \text{mm} \) and the area \( A_{st} \) is assumed to be \( 119 \, \text{mm}^2 \) (the lateral surface of a cylinder with a radius of \( 1.9 \, \text{mm} \) and a length of \( 10 \, \text{mm} \)). The acoustic wall admittances \( Y_w = 1/Z_w \) resulting from equation (4) for the three sets of values are depicted as straight lines in Figure 3 together with the modeling results from [34, Fig. 6] depicted as open circles. Additionally the acoustic admittance of the air enclosed in the canal is depicted in black. Note that in [34] a weighting of the damping constants \( \alpha \) and \( \beta \) wasn’t mentioned, however, the positive phase values at the resonant frequency of \( Y_w \) of their results indicate that they used a damping model which was dominated by mass rather than by compliance (\( \alpha/\omega > \beta/\omega \)). In order to reproduce this effect in Figure 3 a weighting of \( \alpha/\omega = \frac{\beta}{\omega} \) is applied. For the model we propose here, a damping ratio \( \zeta_{st} \) of 0.6 was chosen which leads to modeling results comparable to those in [34] for their baseline set, see Figure 3. Since no reason could be found that the damping is either dominated by mass or by compliance, \( \alpha/\omega = \beta/\omega \) is used in the model proposed here. It should be noted that the choice of a particular damping model is not critical with respect to the resulting immittance values, see discussion in Appendix A.1.

In order to account for the effect of the soft ear canal walls in the sound transmission equation (1) changes to

\[
\begin{bmatrix}
  p_{sc} \\
  q_{sc}
\end{bmatrix} = \begin{bmatrix}
  \left( \prod_i \left[ \begin{array}{c}
  \cosh(\gamma_i \Delta_i) & \sinh(\gamma_i \Delta_i) Z_{twi} \\
  \sinh(\gamma_i \Delta_i) / Z_{twi} & \cosh(\gamma_i \Delta_i)
  \end{array} \right] \right)
\end{bmatrix} \begin{bmatrix}
  1/Z_{w,1} & 0 \\
  1/Z_{tw,1} & 1
\end{bmatrix} \begin{bmatrix}
  p_d \\
  q_d
\end{bmatrix}. \tag{9}
\]

![Figure 3](image-url) Reproduction of the modeling results in [34] of the ear canal wall admittance and admittance of the air in the ear canal using equation (4). The circles correspond to [34, Fig. 6] and the lines correspond to equation (4) using values of \( \rho_{st} = \{1000 \, \text{kg/m}^3, 1100 \, \text{kg/m}^3 \) and \( 1200 \, \text{kg/m}^3 \}, d_{st} = 3 \, \text{mm}, A_{st} = 119 \, \text{mm}^2, E_{st} = \{20 \, \text{kPa}, 210 \, \text{kPa} \) and \( 400 \, \text{kPa} \}, \zeta_{st} = \{0.1, 0.25 \) and \( 0.4 \}, \) and \( \alpha/\omega = \beta/\omega \).

In comparison to models with rigid ear canal walls, the inclusion of the effects of soft ear canal walls will mainly affect frequencies below about \( 1.5 \, \text{kHz} \), see discussion in Section 4 (Fig. 10).

### 2.2.2 Middle ear cavities

The middle ear cavities comprise the tympanic cavity, the aditus ad antrum, the antrum and the mastoid air cells. They are represented by the acoustic impedance \( Z_{cav} \), located between the ear canal two-port and the eardrum within the kernel two-port, see Figure 1. Generally, cavities are modeled using acoustic compliances representing volumes, acoustic masses representing air columns and resistive components introducing damping. The model of \( Z_{cav} \) for adults according to [18] is depicted in Figure 4. It contains the compliance \( N_{cav} \) and the resistance \( W_{cav} \) of the tympanic cavity, the acoustic mass \( M_{ada} \) and the resistance \( W_{ada} \) of the aditus ad antrum, the compliance \( N_{ant} \) of the antrum and the acoustic impedance \( Z_{mac} \) of the mastoid air cells.

With the acoustic compliance of a volume \( V \) given by \( N = V/\rho c^2 \), the resulting acoustic impedance of the middle ear cavities becomes
The acoustic impedance $Z_{\text{cav}}$ results from the pressure $p_{\text{cav}}$ divided by the volume velocity at the eardrum $q_D$.

$$Z_{\text{cav}} = \left( \frac{1}{W_{\text{cav}} + \rho c^2/(j\omega V_{\text{cav}})} + \frac{1}{W_{\text{ada}} + j\omega M_{\text{ada}} + (j\omega V_{\text{ada}}/(\rho c^2)) + 1/\tau_{\text{cav}}} \right)^{-1} .$$  \hspace{1cm} (10)

The acoustic impedance $Z_{\text{mac}}$ of the mastoid air cells was modeled as a series resonant circuit in [18], defined by its resonant frequency $f_{\text{mac}}$, its quality factor $Q_{\text{mac}}$ and its acoustic compliance $N_{\text{mac}}$ associated with the volume $V_{\text{mac}}$ of the air cells,

$$Z_{\text{mac}} = \frac{1 - (\omega/(2\pi f_{\text{mac}}))^2 + j\omega/(Q_{\text{mac}}2\pi f_{\text{mac}})}{j\omega V_{\text{mac}}/(\rho c^2)} .$$  \hspace{1cm} (11)

Significant physiological differences in young infant’s middle ear cavities, compared to those of adults, can be found in the sizes of the tympanic cavity and the antrum and in the status of pneumatization in the mastoid. In [35] the volume of the tympanic cavity was compared between infants aged $\leq$ 1 year and adults. It was found that the average infant’s volume is about 2/3 of the volume in adults (450 mm$^3$/640 mm$^3$). The volume of all middle ear cavities of a 22 days old middle ear was estimated from CT-scans in [36]. The overall volume was estimated to be between 730 mm$^3$ and 930 mm$^3$, with a volume of the tympanic cavity alone of 330 mm$^3$. In the adult’s model, a value of 500 mm$^3$ was used for the tympanic cavity volume and a value of 800 mm$^3$ was used for the antrum [20]. Based on this, values of 330 mm$^3$ for the tympanic cavity and 500 mm$^3$ for the antrum were chosen for young infants in the present model.

According to [37, p. 107] pneumatization of the mastoid starts at the 33rd week of gestation. Starting with very few air cells in the mastoid, the surface of the antrum has an average value of 1 cm$^2$ at birth. In the first year of life, the surface increases due to the pneumatization to 4 cm$^2$. In contrast, the mean adult mastoid air cell surface is about 12 cm$^2$. Because of the weak degree of pneumatization of the mastoid in young infant, the effect is omitted in the model, i.e. $Z_{\text{mac}}$ was removed from the model.

Furthermore, an additional modification in modeling the middle ear cavities is made. As will be shown in Section 3, the influence of the components modeling the aditus ad antrum on the acoustic input impedance at the eardrum of the infant model is small. In order to reduce the overall complexity of the model, these components (namely $W_{\text{ada}}$ and $M_{\text{ada}}$) were omitted.

The resulting model of the middle ear cavities for young infants is depicted in Figure 5.

### 2.2.3 Eardrum, malleus and incus

In [18] the eardrum, the malleus and the incus were aggregated in the kernel two-port, see Figure 1. The kernel comprises the acoustic properties, the transformation from the acoustical to the mechanical domain and the mechanical properties of eardrum, malleus and incus. The equivalent circuit of the kernel is depicted in Figure 6 with the acoustic properties represented by the eardrum acoustic shunt impedance $Z_{\text{ac}}$ on the left side. The two-port in the center models the acousto-mechanical transformation with the effective area $A_D$ of the eardrum. On the right side, the mechanical properties are modeled by the mechanical admittances of the eardrum $y_D$, the malleus $y_m$, the incudomallear joint $y_{mi}$ and the incus $y_i$.

The eardrum acoustic shunt impedance $Z_{\text{ac}}$ was modeled as a series resonator with an additional correction of the phase. The resonator consists of a frequency independent acoustic resistance $W_{\text{ac}}$ and a compliance $N_{\text{ac}}$, and an acoustic mass $M_{\text{ac}}$ increasing with frequency, given by

$$M_{\text{ac}}(f) = M_{\text{ac}0} \left( 1 + \sqrt{\frac{f}{f_{\text{ym}}}} \right) ,$$  \hspace{1cm} (12)

with the acoustic mass $M_{\text{ac}0}$ at $f = 0$ Hz and the characteristic frequency $f_{\text{ym}}$. The eardrum acoustic shunt impedance was given as
$Z_{ac} = \frac{W_{ac} + j\omega M_{ac} + (j\omega N_{ac})^{-1}}{e^{j\phi_Y}},$  \hfill (13)

where

$$\Phi_Y(f) = s_{Yph} \log \left(1 + \frac{f}{f_{Yph}}\right)$$  \hfill (14)

is a phase correction with slope $s_{Yph}$ and cut-off frequency $f_{Yph}$.

In the mechanical part, the admittance of the incudo-malleal joint was defined as

$$y_{mi} = \left(w_{mi} + \frac{1}{j\omega n_{mi}}\right)^{-1},$$  \hfill (15)

with the mechanical resistance $w_{mi}$ and the mechanical compliance $n_{mi}$. The values of the other three mechanical admittances were determined from a common mechanical admittance $y_{dmi}$ of eardrum, malleus and incus.

The equivalent circuit of the common mechanical admittance $y_{dmi}$ comprising parts of the eardrum, the malleus and the incus is depicted in Figure 7.

Herein, $n_{oss}$ and $m_{oss}$ are the mechanical compliance and mass of the ossicles, and $w_{free}$ and $m_{free}$ are the mechanical resistance and mass of the free vibrating part of the eardrum. The coupling between the ossicles and the free vibrating part of the eardrum is represented by the mechanical compliance $n_{cpl}$ and resistance $w_{cpl}$. The mechanical admittance of the incus were then determined using

$$y_i = \frac{1}{3} y_{dmi},$$  \hfill (16)

and the mechanical admittance of the eardrum and the malleus using

$$y_{dm} = \left(1/y_d + 1/y_m\right)^{-1} = \frac{2}{3} y_{dmi},$$  \hfill (17)

according to [20].

The acoustical to mechanical transformation as described in [19] is achieved by a gyrator two-port with a frequency-dependent, complex-valued effective eardrum area $A_D$. Specifically, $A_D$ is given by a second order low-pass filter $A_D'$ with an additional correction term $\Phi_A$ for the phase at higher frequencies, resulting in

$$A_D = A_D' e^{\Phi_A}. $$  \hfill (18)

The low-pass filter was defined as

$$A_D' = \left(A_0 + A_\infty \frac{1 - (\omega/(2\pi f_A))^2 + j\omega/(Q_A 2\pi f_A)}{A_\infty}\right)$$  \hfill (19)

with the eardrum area $A_0$ at low frequencies $\omega$, the area $A_\infty$ for frequencies $\omega \rightarrow \infty$, the low-pass resonant frequency $f_A$ and the low-pass quality factor $Q_A$. The phase correction term was defined as

$$\Phi_A = \begin{cases} 
0 & \text{if } \omega < 2\pi f_{Aph} \\
 s_{Aph} \log \left(\frac{\omega}{2\pi f_{Aph}}\right) + \arg \{A_D(f_{Aph})\} & \text{if } \omega \geq 2\pi f_{Aph}
\end{cases}$$  \hfill (20)

with the slope $s_{Aph}$, the cut-off frequency $f_{Aph}$ and the phase angle of $A_D'$ at $f_{Aph}$ arg $\{A_D'(f_{Aph})\}$.

In the present modeling approach, besides modifications that pertain to differences between infants and adults, two simplifications were made that are not related to differences between infants and adults. First, in [19], the parameters used to model the eardrum acoustic shunt impedance were derived from acoustic measurements, but they were not directly related to physiological properties. As explained
above, basically a series resonator was used to model $Z_{ac}$, with a phase correction term comprising 3 additional parameters used to modify the impedance at high frequencies. The first one, $f_{ph}$, causes a steeper slope of the acoustic impedance. The other two parameters, $N_{ph}$ and $f_{ph}$, change the phase response from approaching $-90^\circ$ at high frequencies to positive phase values. A similar effect as caused by these three parameters can occur if an additional acoustic mass, acting in parallel to the resonator, e.g. a small air column between the acoustic impedance measurement device and the object to be measured, is introduced. In [19] the authors themselves remark that their measurement results are questionable at high frequencies (10 kHz). They explicitly refer to a minimum in the measured acoustic admittance ($1/Z_{ac}$) which is not considered in their model. However, it would then be consequent to ignore the phase increase as well.

Therefore, in the present approach, the phase correction term is omitted. Thus equation (13) simplifies to

$$Z_{ac} = W_{ac} + j\omega M_{ac} + (j\omega N_{ac})^{-1},$$

(21)

Note that all parameters in equation (21) ($W_{ac}$, $M_{ac}$ and $N_{ac}$) have values which do not depend on frequency. In order to be able to adapt at least the acoustic mass and the acoustic compliance to physiological conditions, the model of a stretched circular membrane according to [38, pp. 71, 225] is used. The acoustic compliance is then given by

$$N_{ac} = \frac{\pi}{8} \frac{a_{ed}^2}{T_{0,ed} h_{ed}},$$

(22)

with $a_{ed}$ the radius of the eardrum, $T_{0,ed}$ the eardrum tension and $h_{ed}$ the thickness of the eardrum. The acoustic mass is given by

$$M_{ac} = \frac{4}{3} \frac{\rho_{ed} h_{ed}}{\pi a_{ed}^2},$$

(23)

with $\rho_{ed}$ the membrane mass density.

The second simplification not related to differences between infants and adults is made in the effective eardrum area $A_D$. As will be shown in Section 3, the influence of the three parameters high frequency eardrum area ($A_{ac}$), slope of the phase correction term ($f_{ph}$) and cut-off frequency of the phase correction term ($f_{ph}$) on the resulting acoustical eardrum impedance $Z_D$ is very small. In order to reduce the overall complexity of the model, these parameters are omitted. The simplified effective eardrum area is then given by

$$A_D = \frac{A_0}{1 - (f/f_{ac})^2 + jf/(Q_{ac}f_{ac})}.$$  

(24)

Physiological differences of the eardrum between young infants and adults were reported in [39]. It was found that the thickness of the pars flaccida (Shrapnell’s Membrane) decreases with age, mainly during the first year of life. At an age of about 4 month the pars flaccida is about 3 times thicker compared to the thickness at an age of 21 years. Assuming the area of the pars flaccida is about 0.2 of the overall eardrum area, the parameters of acoustic and mechanic masses of the eardrum are multiplied by a factor of $3 \cdot 0.2 + 0.8 = 1.4$. According to equation (22), the acoustic compliance $N_{ac}$ is inversely proportional to the thickness of the membrane and is therefore divided by 1.4.

Age-related changes of the ossicles were reported in [40], where the average weight of malleus and incus is roughly 25% higher in adults than in the age group from 0 to 10 months. Since a reduction of the ossicles’ mechanical mass in the model by 25% has almost no effect on the acoustical eardrum impedance, this age-related difference is neglected.

There are some parameters in the model for which a direct physiological link is not obvious. This applies to the damping and to the low-pass used to model the effective eardrum area. By comparing the acoustic impedance predicted by the model with impedances measured in infant’s ear canals from [41], it was found that an adaption of three more parameters was necessary. Firstly, the resistance $W_{ac}$ of the eardrum acoustic shunt impedance was reduced by a factor of 0.7, secondly, the low-pass resonant frequency $f_{ph}$ of the effective eardrum area was increased by a factor of 2 and thirdly, the quality factor $Q_A$ of the effective eardrum area was reduced to 1.

### 2.2.4 Stapes and cochlea

In [18] the stapes and the cochlea were represented by the mechanical admittance of the stapes $y_{st}$ and the acoustic input impedance of the cochlea $Z_C$. Both were modeled by simple vibrators. The mechanical admittance of the stapes was given by

$$y_{st} = \left( \frac{a_{st}}{1 + j\omega m_{st}} + j\omega m_{st} \right)^{-1},$$

(25)

and the acoustic input impedance of the cochlea was given by

$$Z_C = \left( \frac{a_{st} + j\omega m_{ac} + j\omega m_C}{A_{stF}} \right)^{-1},$$

(26)

with the respective mechanical elements resistance $a_{st}$, compliance $m$ and mass $m$ and with the area of the stapes footplate $A_{stF}$. The compliant element of $y_{st}$ represents the annular ligament and the compliant element of $Z_C$ is mainly caused by the round window membrane, see [20].

To our knowledge no physiological differences in stapes and cochlea input between infants and adults have been reported in literature. Therefore, we propose to retain the original model from [18] for this part of the overall model. However, it should be mentioned that in [29] it was stated that the developmental changes in the size of the middle ear cavities may be associated with a change in ossicular orientation.

### 3 Effect of proposed model adaptations on the eardrum impedance

In the previous section, several adaptations were proposed with respect to the model according to [18]. In this
4 Ear canal impedance of the proposed model

To give insight into how the ear canal impedance $Z_{ec}$, which is the quantity measured in a WAI-measurement, of the newly proposed model for infants differs from the original model according to [18] for adults, some exemplary results are shown in Figure 10. For both, infant and adult, the ear canal is modeled assuming a constant cross section. Using the values of ear canal geometry given in Section 2.2.1 with a probe insertion depth of 5 mm, which has to be subtracted from the ear canal length, for the infant’s ear canal a length of 11 mm and a radius of 2.7 mm is assumed while for the adults’ ear canal a length of 18 mm and a radius of 5.2 mm is assumed. In addition to the ear canal impedance of the infant’s model ($Z_{ec, \text{infant}}$) the impedance of the same model neglecting the effect of compliant ear canal walls ($Z_{ec, \text{infant rigid ec walls}}$) is shown.

By comparing the ear canal impedances of the infants’ model with and without compliant walls, it can be seen that the effect of compliant ear canal walls reduces the magnitude by several dB up to about 500 Hz. A substantial difference in the phase of $Z_{ec}$ can be seen up to about 1.5 kHz. At higher frequencies, the effect of compliant ear canal walls can be ignored, provided our proposed model is valid. A comparison of $Z_{ec}$ with $Z_D$ shows for the adult model that the magnitude of the eardrum impedance $Z_D$ is high compared to that of the ear canal impedance. For the infant model, this is only true at frequencies up to about 1 kHz and above about 4 kHz. Conversely, between about 1.5 kHz and 3 kHz, $Z_{ec}$ is governed by $Z_D$.

In Figure 11, the effect of varying either the ear canal radius (left) or the ear canal length (right) by ±50% can be seen. In both cases the ear canal volume varies, resulting in an increased low-frequency impedance magnitude with decreased volume. While a decreasing ear canal radius increases the impedance magnitude at higher frequencies, too, a decreasing ear canal length shifts the local extrema to higher frequencies.

The effects of varying the parameters modeling the compliant ear canal walls by ±50% on the ear canal impedance $Z_{ec}$ are depicted in Figure 12. As already seen above, substantial differences in $Z_{ec}$ are restricted to frequencies below about 1.5 kHz. As can be seen on the left side, increasing Young’s modulus $E_{sat}$ shifts the magnitude minimum and the phase maximum to higher frequencies. As expected, increasing the damping ratio $z_{sat}$ results in a shallower magnitude minimum and phase maximum. On the right side it
can be seen that increasing the thickness $d_s$ shifts the magnitude minimum and the phase maximum to lower frequencies.

In summary, for the infant ear, the main characteristics of the ear canal impedance $Z_{ec}$ and their underlying structures are as follows: At frequencies below about 1 kHz, the compliance of the air volume resulting from the ear canal geometry, together with the compliant ear canal walls and the compliant eardrum, determine the level of the magnitude. Additionally, the vibrating ear canal walls cause a magnitude minimum or plateau and a maximum in phase around 400–700 Hz. Around 1.8 kHz, the acoustic input impedance at the eardrum causes a magnitude minimum with a phase increasing with frequency. At high frequencies above about 3 kHz, the ear canal geometry causes a magnitude minimum with a phase increasing with frequency.

5 Experimental validation

In this section, the model predictions of the ear canal impedance will be compared with measurements published in [41]. As described in [41], the measurement method is based on [44], extended by consideration of discontinuity and end corrections in [22]. The method including the calibration procedure is described in detail in [22]. The measurements were performed using a custom-made impedance probe to measure the ear canal impedance on infant ears. The probe was designed to avoid over-pressure in the ear canal caused by inserting it into the ear canal at the cost of a decreased signal-to-noise ratio (SNR) at frequencies below about 1 kHz, further details can be found in [41]. This was realized by a pressure equalizing duct with an inner diameter of 0.6 mm. The subjects who participated in the study were aged between 2 weeks and 5 months. It should be noted that the infant ear undergoes developmental changes within the age range of the participants, i.e. an age-related effect is contained in the measurement data. The middle ear status of the participants was assessed by ENT-doctors specialized in pediatric audiology to be either normal, pathological or unclear at the time of testing. The assessment was based on the results of 1 kHz tympanometry, OAE- and/or AABR-screening, and ear-microscopy, see again [41] for details. Note that in the following only the measurement data of the ears assessed to have a normal middle ear status are used. This comprised 30 ears from 24 infants in total.

In Figure 13, the measured ear canal impedances of 30 ears (pale colored lines), the modeled ear canal impedance, again using a constant ear canal cross section with 11 mm length and 2.7 mm radius (dashed black line) are depicted. Additionally, the modeled ear canal impedance using a constant ear canal cross section with 14 mm length and 1.7 mm radius is depicted (straight black line). All default parameter values used for the predictions are given in Appendix A.2. Since the impedance measurements in [41] suffer from a bad SNR at low frequencies, due to noise generated by the subject during the measurement, only those impedance values are depicted in which the coherence between the signal applied to the probe speaker and the signal sensed by the probe microphone exceeded 0.5. Another effect that appeared in some measurements was acoustic leakage, which can be seen by phase values around 0 or larger below about 1 kHz. Although it would be easily possible to incorporate acoustic leakage into the model, this wasn’t done here in order not to increase the number of parameters further.

Comparing the ear canal impedance (dashed line) of the model (using $a_{ec} = 2.7$ mm and $l_{ec} = 11$ mm) with those of the measurements, Figure 13 shows that the overall course of the impedance, with its main characteristics, is in agreement in both, magnitude and phase. At low frequencies up to about 1 kHz, the effect of compliant ear canal walls as described in Section 4 can be seen in several measurements. The first minimum of the impedance magnitude at about 1.8 kHz is also present in the measurements, however, most measurements have a larger value compared to the model. Model and measurements have a phase increasing with frequency between 1.2 kHz and 2.4 kHz, but in the model, the slope of the phase is smaller compared to the measurements. At higher frequencies, inter-individual differences in the measurements are quite large, but a significant difference between the measurements and the model is that the minimum in the impedance magnitude of the model is much
steepener and the phase between about 2.5 kHz and 8 kHz lie below the measurements.

The high-frequency-minimum in the impedance magnitude is only caused by the ear canal dimensions, as can be seen in Figure 11. If the ear canal geometry is slightly changed, the agreement between model and measurement can be significantly increased. $Z_{ec}$, depicted as straight line in Figure 13, is modeled with an ear canal radius of 1.7 mm and an ear canal length of 14 mm. As can be seen, the agreement with the measurements is much better at high frequencies and, furthermore, also between 1.2 kHz and 2.4 kHz, in both magnitude and phase. However, between 3.5 kHz and 7 kHz, the model phase still lies below the measurements.

6 Influence of ear canal shape

So far, for all calculations of the impedance $Z_{ec}$, a cylindrical ear canal shape was assumed. As described in Section 2.2.1, it is also possible to compute $Z_{ec}$ for varying cross-sectional area functions of the ear canal. In Figure 14 the impedance of the ear $Z_{ec}$ is depicted for three different ear canal shapes chosen to cover the range of possible shapes. From lateral to medial these shapes are: a cylindrical canal, a canal with linearly decreasing cross-sectional area, and a canal with linearly increasing cross-sectional area. All ear canals have the same volume and the same length.

As can be seen, compared to the cylindrical canal, a conical shape with decreasing cross-sectional area shifts the magnitude minimum to higher frequencies. If the cross-sectional area increases, the minimum is shifted to lower frequencies.

Changes in the ear canal shape without changing the ear canal volume mainly affect $Z_{ec}$ at high frequencies. In the medium frequency range where $Z_{ec}$ is dominated by $Z_D$, only a small effect can be observed for the magnitude and a slightly larger effect can be observed for the phase of $Z_{ec}$ in the 2-3 kHz frequency range. Real ear canals will differ from the model by shapes that deviate from circular cross sections and by curvature. The effects shown above indicate that real ear canal shapes will have a strong influence on $Z_{ec}$ at high frequencies, but at medium

Figure 10. Eardrum impedances $Z_D$ (left) and input impedances of the ear $Z_{ec}$ (right) resulting from the model according to [20] and our model adapted to infant’s ears.
frequencies below about 3 kHz the influence on $Z_{ec}$ will be rather small.

7 Comparison to similar models

One of the first models considering the effect of soft and flexible ear canal walls of young infants on WAI-measurements was proposed in [4]. In that study, the authors presented measurement-based acoustic input impedances for different age groups. They used an electro-acoustic approach to model the acoustic input impedance $Z_{ec}$ for the age group of 1 month in the frequency range from 125 Hz to 2 kHz. The ear canal walls were represented by a resonator with fixed values of the resonant frequency ($f_0 = 450$ Hz), the quality factor ($Q = 2$), and the compliance of the wall ($N_w = 1.6 \cdot 10^{-12}$ m$^3$/Pa s)). Compared to the model of the present study, provided that comparable ear canal dimensions are used, the effect of the ear canal wall on $Z_{ec}$ is very similar at frequencies below the resonant frequency, see Figure 15. At higher frequencies, there are larger differences, mostly because at these frequencies the acoustic input impedance at the eardrum $Z_D$ becomes dominant, which has a smaller magnitude in the present model. In contrast to the present modeling approach, beside being restricted to frequencies below 2 kHz, in [4] neither a relation between ear canal dimensions and compliance of the wall, nor other relations to physiological properties are given.

In [45], a finite-element model of a 22 days old newborn’s ear canal was presented. The purpose of that model was to study the volume change in the ear canal under different static pressures of air as applied in tympanometry. The authors showed that using a Young’s modulus of the soft tissue in the range from 30 kPa to 90 kPa and a Poisson’s ratio of 0.475 resulted in plausible displacements and volume changes. The values used in the present model are in that range (90 kPa for Young’s modulus) or very close since an incompressible medium is assumed (i.e. Poisson’s ratio of 0.5).

Another finite-element model of the same ear as in [45] was published in [34]. In that study, the sensitivity of the acoustic input admittance of the ear to changes in material properties was investigated up to frequencies of 2 kHz. Finally, a model with material properties adjusted to match some clinical data was proposed. A refined version of the model, in which the frequency range was extended up to 10 kHz, was proposed in [46]. Compared to the model of the present study, their first version model results in similar predictions, see Figure 15. Between about 500 Hz to 1.5 kHz the impedance magnitude of their model is about 3 dB greater. At about 800 Hz the phase responses start to deviate resulting in a difference of $46 \degree$ at 2 kHz ($-33 \degree$ with the model in [34] versus $13 \degree$ with the model of the present
The majority of the measurements depicted in Figure 15 have a positive phase value at 2 kHz and hence agree better with the model proposed here. The prediction of their refined model is closer to that of the present study, especially in the frequency range from 1 to 2 kHz, as can be seen in Figure 15. At higher frequencies some substantial differences can be observed, in particular, at the magnitude maximum at 3.6 kHz and the phase maximum at 3 kHz. At these frequencies $Z_{ec}$ transitions from dominance by $Z_D$ to dominance by the ear canal geometry. Compared to the measurements, the prediction of the present study tends to the maximum values measured and the prediction according to [46] tends to the lower values measured, however both predictions are within the range of measurements at frequencies between 2.4 kHz and 4 kHz. In the model according to [46], a resonance at about 6 kHz and a local minimum at 7 kHz can be seen which are not predicted with the model of the present study. In [46] it was explained that the resonance is caused by the first mode of the middle ear cavity and the minimum is caused by the first standing wave mode in the ear canal. A resonance in the middle ear cavities at about 6 kHz, but much less pronounced, could be predicted by the model of the present study if a value of 600 kg/m$^4$ for the acoustic mass of the aditus ad antrum $M_{ada}$ would be used. However, a comparison with the measurements doesn’t show an effect of much importance, therefore, omitting $M_{ada}$, as proposed in Section 2.2.2, seems to be justified. The magnitude minimum at 7 kHz could be reproduced, but it would require an ear canal length of at least 16 mm which doesn’t match the ear canal dimensions given in [34]. In summary, up to about 6 kHz the predictions of both models are comparable. However, the parametric model proposed here is easier to adapt to deviating properties, caused e.g. by inter-individual differences, than the single-case finite-element model from [46].

8 Discussion

Based on an existing model for a adults, a parametric electro-acoustic model has been developed, allowing predictions of acoustic input impedances of young infants’ ears. Special attention has been paid to using model parameters representing physiological conditions whenever possible, and to reducing the number of parameters in comparison to the original model for adults according to [18–20]. Predictions from the model showed that for frequencies below about 1 kHz, the ear canal impedance $Z_{ec}$ is determined by the volume of air in the ear canal and by the compliant ear canal walls and that of the eardrum. At a medium frequency range around 1.8 kHz, $Z_{ec}$ is determined by the eardrum impedance $Z_D$, and at high frequencies (above about 3 kHz) $Z_{ec}$ is determined by the ear canal...
geometry. A comparison with impedances measured on infant’s ears showed a good agreement in the main characteristics.

8.1 Current limitations of the model

The proposed model is developed to predict acoustic input impedances of the ears of young infants aged younger than 6 months. Known age-dependent physiological differences have been considered in the model, these differences are: (1) the soft and flexible ear canal walls which will get acoustically rigid with increasing age. (2) Pneumatisation of the mastoid, which increases with age and can be neglected for very young infants. (3) The volume of the tympanic cavity and the antrum, which also increases with age. (4) The thickness of the pars flaccida of the tympanic membrane, which decreases with age. It would be desirable to have the possibility of a continuous age-dependent adaptation of the model. However, this would require a more detailed knowledge about the developmental processes, which is not available at present.

In the proposed model, the ear canal shape is restricted to consecutive cylindrical segments with different cross sections. Real ear canal shapes will deviate from circular cross sections and will have curvatures. It is assumed that these differences may have a noticeable effect on the acoustic input impedance of the ear \( Z_{ec} \) at high frequencies (\( \geq 4 \text{ kHz} \)), but only very small effects at medium frequencies. However, at high frequencies, many of the measured \( Z_{ec} \) showed magnitudes larger than those predicted by the model. A higher magnitude of \( Z_{ec} \) in that frequency range means that the impact of \( Z_D \) on \( Z_{ec} \) is larger. The consequence for model predictions of the impact of specific middle ear conditions on immitance measurements is that the influence of specific middle ear conditions may not only be detectable at medium frequencies, but at higher frequencies as well.

The proposed model is currently limited to predictions at ambient pressure in the ear canal, whereas many WAI test are performed over a tympanometric range of air pressures in the ear canal. Nevertheless, the model is potentially extendable to account for differences in air pressure between ear canal and middle ear, by linearizing it to different static pressure states, as will be discussed in the companion paper [47].

Although the structure of the model would permit predictions of middle ear transfer functions, such predictions weren’t investigated. The aim of the proposed model is to support WAI-measurements by permitting statements about the acoustic input impedances \( Z_{ec} \) and \( Z_D \). A verification of the model for the prediction of middle ear transfer functions...
impedances would require considerably different reference data. This may be part of future work.

8.2 Differences to similar models

The proposed model confirms that there are large differences in the acoustic input impedances of ear canal and eardrum between adults and young infants. Very few models for young infant ears exist. The electro-acoustic modeling approach in [4] was intended to simulate the effects of the soft and flexible ear canal walls of young infants. The predicted effects are somewhat comparable to those predicted by the model of the present study. However, the parts of the model in [4] representing the ear canal geometry and the eardrum are very basic and not related to physiological properties. In contrast, the model parameters of the present study are very close to physiological properties allowing predictions for different physiological conditions. On the other hand, finite-element models of young infant ears exist [34, 36, 46]. The predictions of these models are very similar to those of the present model. The model proposed in [46] allows predictions up to 10 kHz. Compared to the model of the present study, properties of ear canal shape can be reproduced much better with the finite-element model. However, predictions of the effects resulting from individual differences or different middle ear conditions would be extremely costly.

9 Conclusions based on the model

Based on the parametric model of young infants’ eardrum and ear canal impedances proposed in this paper, the following conclusions can be made:

- The soft and flexible ear canal walls in young infant ears affect the measured acoustic input impedance of the ear $Z_{ec}$ up to about 1.5 kHz, no significant effect is expected at higher frequencies.
- At a medium frequency range around 1.8 kHz, the measured input impedance $Z_{ec}$ is dominated by the eardrum impedance $Z_D$. Therefore, it can be expected that different middle ear conditions will have the largest effect on $Z_{ec}$ in the medium frequency range.
- At high frequencies, the model predictions indicate that $Z_{ec}$ is dominated by ear canal properties and therefore, it might not be possible to detect different middle ear conditions at high frequencies. However, predictions must be taken with caution at these frequencies because many measurements showed higher $Z_{ec}$ magnitudes which could mean that in some cases middle ear conditions could still be detected.

In part II of this paper, the model will be extended to predict the influence of different pathological middle ear conditions on the acoustic input impedances of the ear canal and the eardrum of young infants in order to investigate implications for WAI-measurements.

Conflict of interest

The authors declare no conflict of interest.

Acknowledgments

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Data availability statement

An implementation of the complete model is available on GitHub, under the reference https://github.com/tobiassankowsky/acoustic_impedance_infant_ear.

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Appendix A

A.1 Notes on damping models for compliant ear canal walls

In order to account for the damping caused by the soft and flexible ear canal walls, in the present study, each slice i of the ear canal has a complex impedance \( Z_{nc} \) in parallel (see Section 2.2.1). \( Z_{nc} \) is a series resonator with an acoustic mass, an acoustic compliance and an acoustic resistance. The acoustic resistance is modeled using a frequency independent damping ratio \( \zeta_m \) (equation (7)) with damping constants \( x \) and \( \beta \). The modeling is similar as in [34, 46], however, a significant difference is that in the present study the damping constants are chosen such that \( x / \omega = \beta \omega \) with the result that the undamped natural frequency equals the damped resonant frequency of \( Z_{nc} \), i.e. the minimum of the impedance magnitude coincides with a phase value of 0° (see left side of Fig. 16 black line). In [34, 46], by contrast, the phase value of \( Z_w \) at the magnitude minimum is negative, i.e. \( x / \omega > \beta \omega \) or rather the resistive damping is dominated to a greater extent by mass than by compliance (see left side of Fig. 16 red line). Another modeling approach is used in [4] in which \( Z_w \) is modeled by a resonator consisting of an acoustic mass in series with a resistance and a compliance in parallel. The resulting effect is that the resistive damping is dominated only by the compliance (see left side of Fig. 16 red circles).

The input impedance of the ear \( Z_{nc} \) of the different models is depicted on the right side of Figure 16. For comparisons, \( Z_{nc} \) resulting from the model of the present study assuming rigid ear canal walls is depicted, additionally. A noticeable difference caused by the different approaches used to model \( Z_w \) results only in the phase of \( Z_{nc} \) at very low frequencies (<400 Hz) with the model according to [4] compared to the other models. Note that the difference at about 500 Hz is mainly caused by the input impedance of the eardrum \( Z_D \) which is assumed to be larger in [4] compared to the other models.

It can be concluded that the effect of the different approaches which have been proposed to model the resistive component of the damping caused by the soft and flexible ear canal walls do not show a substantial different impact on \( Z_{nc} \). If at all, differences are limited to the impedance phase at frequencies below about 400 Hz.
A.2 Default parameter values

**Table A1.** Default values of ear canal parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Infant value</th>
<th>Adult value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{ec}$ (or widths $\Delta_i$)</td>
<td>14 mm</td>
<td>18 mm</td>
</tr>
<tr>
<td>$a_{ec}$ (or areas $A_i$)</td>
<td>1.7 mm</td>
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</tr>
<tr>
<td>$g_{ec}$</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>$E_{st}$</td>
<td>0.6</td>
<td>–</td>
</tr>
<tr>
<td>$\rho_{st}$</td>
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</tr>
<tr>
<td>$d_{st}$</td>
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</tr>
<tr>
<td>$d_{st}$</td>
<td>3 mm</td>
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**Table A2.** Default values of middle ear cavities parameters.

<table>
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<tr>
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<th>Adult value</th>
</tr>
</thead>
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<tr>
<td>$f_{mac}$</td>
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</tr>
<tr>
<td>$Q_{mac}$</td>
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<td>0.4000</td>
</tr>
<tr>
<td>$V_{mac}$</td>
<td>–</td>
<td>8 cm$^3$</td>
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<tr>
<td>$M_{ada}$</td>
<td>–</td>
<td>880 kg/m$^4$</td>
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<tr>
<td>$W_{ada}$</td>
<td>–</td>
<td>17e5 Pa s/m$^3$</td>
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<tr>
<td>$V_{ant}$</td>
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<td>0.8 cm$^3$</td>
</tr>
<tr>
<td>$V_t$</td>
<td>0.33 cm$^3$</td>
<td>0.5 cm$^3$</td>
</tr>
<tr>
<td>$W_t$</td>
<td>2e6 Pa s/m$^3$</td>
<td>2e6 Pa s/m$^3$</td>
</tr>
</tbody>
</table>
Table A3. Default values of middle ear kernel parameters.

<table>
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<th>Adult value</th>
<th>acc. to [20] (adult)</th>
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<tbody>
<tr>
<td>$a_{\text{ed}}$</td>
<td>4.3702 mm</td>
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</tr>
<tr>
<td>$T_{0,\text{ed}}$</td>
<td>3.6465e+05 Pa</td>
<td>3.6465e+05 Pa</td>
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</tr>
<tr>
<td>$h_{\text{ed}}$</td>
<td>0.112 mm</td>
<td>0.08 mm</td>
<td>–</td>
</tr>
<tr>
<td>$\mu_{\text{ed}}$</td>
<td>1100 kg/m$^3$</td>
<td>1100 kg/m$^3$</td>
<td>–</td>
</tr>
<tr>
<td>$(M_{\text{ac})}^a$</td>
<td>2737.8 kg/m$^4$</td>
<td>1955.6 kg/m$^4$</td>
<td>2400 kg/m$^4$</td>
</tr>
<tr>
<td>$(N_{\text{ac})}^a$</td>
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<td>4.9102e−12 m$^2$/N</td>
<td>5e−12 m$^2$/N</td>
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<td>1500 Hz</td>
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<td>0.02 N s/m</td>
<td>0.02 N s/m</td>
</tr>
<tr>
<td>$n_{\text{ph}}$</td>
<td>0.0005 N/m</td>
<td>0.0005 N/m</td>
<td>0.0005 N/m</td>
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<tr>
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<td>0.08 N s/m</td>
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<tr>
<td>$m_{\text{os}}$</td>
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<td>7e−06 kg</td>
<td>7e−06 kg</td>
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<tr>
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<td>0.003 m/N</td>
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<td>4e−05 m/N</td>
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<td>$m_{\text{inf}}$</td>
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</tbody>
</table>

$a$ Not used as a parameter in the adapted model (see equations (23) and (22), values are given for comparisons to the model according to [20].

Table A4. Default values of stapes and cochlea parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Infant value</th>
<th>Adult value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\text{St}}$</td>
<td>3e−06 kg</td>
<td>3e−06 kg</td>
</tr>
<tr>
<td>$n_{\text{St}}$</td>
<td>0.0012 m/N</td>
<td>0.0012 m/N</td>
</tr>
<tr>
<td>$w_{\text{St}}$</td>
<td>0.018 N s/m</td>
<td>0.018 N s/m</td>
</tr>
<tr>
<td>$m_{\text{C}}$</td>
<td>1e−05 kg</td>
<td>1e−05 kg</td>
</tr>
<tr>
<td>$n_{\text{C}}$</td>
<td>0.011 m/N</td>
<td>0.011 m/N</td>
</tr>
<tr>
<td>$w_{\text{C}}$</td>
<td>0.07 N s/m</td>
<td>0.07 N s/m</td>
</tr>
<tr>
<td>$A_C$</td>
<td>3 mm$^2$</td>
<td>3 mm$^2$</td>
</tr>
</tbody>
</table>