



Rayleigh angle incident ultrasonic beam shape design influence on reflected beam

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Abstract – Current evolutions in transducer design, such as phased arrays, but more importantly, metamaterials-based acoustic lenses, potentially enable generating specific beam shapes earlier unconsidered. It is known that the Schoch effect, when a bounded incident beam on a submerged solid reflects at the Rayleigh angle, depends on the beam width and the frequency. This work numerically explores the consequence of the shape of such beams on the Schoch effect and invites further experimental work. The study investigates square shapes and beams with exponential flanks compared to Gaussian reference profiles and incorporates diffraction upon sound propagation to resemble reality better. It is shown that stunning differences occur depending on the beam shape, particularly for square beams.

Keywords: Schoch effect, Metamaterials, Rayleigh angle, Beam shape, Fourier method

1 Introduction

The research fields of acoustics and optics have developed simultaneously. In particular, acoustics was boosted by advances in optics since the development of the laser; while the interaction of light with sound, acoustic-optics, first experimentally proven by Debye and Sears [1], and Lucas and Biquard [2] in 1932, gained momentum in the 1960 s. It's for instance, at the World exposition of 1958 that the Belgian school of R. Mertens became involved in this field [3–5] after the group of Hiedemann, originating from the group of James Franck, had made significant progress even before the arrival of a practical laser device [6–12]. When the acoustic counterpart of the optical Goos–Hänchen effect was searched for, which constituted a forward lateral light beam displacement internally reflected from a dielectric interface [13], it was discovered for sound beams incident from a liquid on a solid, at the Rayleigh angle. The latter is nowadays referred to as the Schoch effect. Later, the phenomenon was equally observed for Lamb angles of the sound incident on thin plates.

The effect of the lateral ultrasonic beam shift was studied by Tamir and Bertoni [14] and later also others, such as Breazeale and Adler [15–17]. It was called the Schoch effect, after the person who was the first to observe it experimentally [18–20]. The phenomenon may also occur in the solid, as is known from experience and was also reported earlier on by Kutzner [21], Filippov [22] and others

[23]. More sophisticated materials such as dielectric materials or phononic crystals may also exhibit the effect [24, 25]. The phenomenon is often accompanied by a null strip in between a specular and a nonspecular reflected beam. The expression “null strip” was first mentioned in Neubauer and Dragonet [26].

Figure 1 shows a typical Schlieren set up as we use to experimentally illustrate the Schoch effect as in Figure 2. Light passes from a laser source, through a sound field, to a projection screen only if it is not blocked by a spatial filter, which is only possible if it is diffracted by sound. Therefore, Schlieren photography enables visualization of sound.

Figure 2 shows the Schoch effect for a bounded beam incident on a thin aluminum plate at a Lamb angle and illustrates the effect as it occurs in experiments. For thick plates, the same phenomenon is visible at the Rayleigh angle. The author made the image, using Schlieren photography, essentially based on the acousto-optic effect [27], as illustrated in Figure 1.

A cross-fertilization between optics and acoustics in more recent times is found in the similarity between photonic crystals and phononic crystals or metamaterials in general. Recent advances in metamaterials [28–38] offer novel possibilities in transducer design, beam forming, and beam focusing. Provided with a new range of design possibilities, potential interest transpires in ultrasonic non-destructive testing and the occurrence of a guided wave-related beam effect, such as the Schoch effect, under unorthodox shapes of incident beams.

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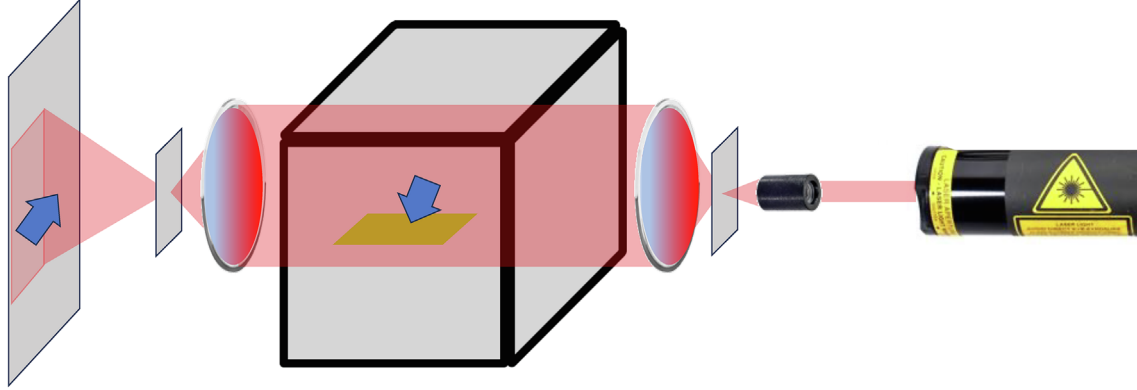


Figure. 1. Schematic of a Schlieren set up. Coherent light passes from right to left: HeNe laser source; microscopy lens to focus light onto a pinhole; metal disk with transparent pinhole; converging lens to make light parallel; water tank in which the sample (yellow) is placed and is impinged by ultrasound (blue arrow); lens to converge light to a spatial filter (e.g. a black ink spot on a transparent glass plate, which blocks undiffracted light); projection screen on which the sound beam (blue arrow, formed by diffracted light not blocked by the spatial filter) becomes visible and can be photographed.

In what follows, particular beam shapes are investigated, and the effect of their shape on the Schoch effect is simulated and discussed.

2 Studied beams and material properties

Plane waves incident from a liquid on a solid generate reflected and transmitted waves that obey Snell's law and allow the determination of angles of incidence causing guided waves, such as leaky Rayleigh waves on a thick solid or leaky Lamb waves in a thin solid plate. The leakage field from the guided wave into the liquid interacts with specularly reflected sound, causing interference in space and time. When incident sound is not a plane wave but a bounded beam, the interference pattern sometimes consists of the appearance of a resultant reflected sound field exposing a beam shift and sometimes even the appearance of two reflected beams with a null zone in-between. Studies usually consider incident *Gaussian beams*. However, it may be interesting to investigate other beam shapes numerically before experimenting in the framework of the recent development of metamaterial-based acoustic lenses. The reflected beam effects can be applied to re-shape beams for particular applications or to change the efficiency with which incident sound is transformed into guided waves.

The study applies bounded harmonic beams and does not consider pulses. Viscous damping effects or nonlinearity are also not considered. As shown earlier, when averaged over time, pulses do not change Schoch effect characteristics on thick plates but prevent a Schoch effect on thin plates, which is not the scope of this work [39]. Here we do not study the appearance or absence of the effect but the beam shape's influence. For a sound beam $\psi(x, z, t)$, with spatial profile $\phi(x, z)$, we consider three cases for $\phi(x, z)$, namely a *Gaussian beam* $G(x, z)$, a beam $C(x, z)$ with exponential flanks as the roof of a Japanese temple, and a *square beam* $P(x, z)$. A parameter W is used for the Gaussian beam width. A parameter β is used for the exponential behavior

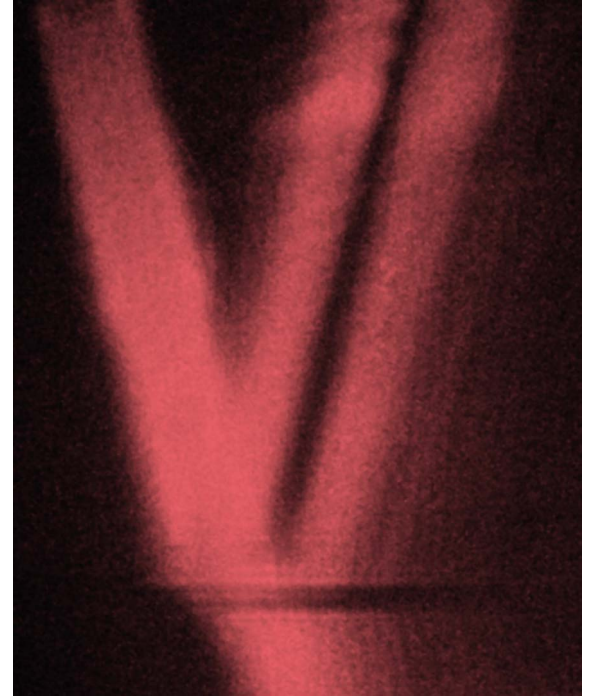


Figure 2. Example of a Schlieren image, taken using a HeNe laser, of a bounded beam incident (from the top left) onto a thin Aluminum plate. In this example, the Schoch effect with null-strip is caused by a leaky Lamb wave and is visible in reflection. In transmission, the effect is equally visible, though without null-strip. A similar effect occurs for thick plates at the Rayleigh angle.

in the shape of the beam C . Finally, a parameter B is used to determine the width of the square profile.

In particular, a *Gaussian beam* $G(x, z)$ is characterized by its profile at $z = 0$ as,

$$G(x, 0) = \exp\left(-\left(\frac{x}{W}\right)^2\right). \quad (1)$$

A beam $C(x, z)$ has the shape of the roof of a Japanese temple, hence the expression *Japanese temple beam* further in this work,

$$C(x, 0) = \exp(-\beta|x|). \quad (2)$$

Finally, a *square beam* $P(x, z)$ is defined as,

$$P(x, 0) = \exp\left(-\left(\frac{x}{B}\right)^{64}\right). \quad (3)$$

To compare the different beams with one another, we demand that they possess the same amount of energy,

$$\int_{-\infty}^{+\infty} G^2 dx = \int_{-\infty}^{+\infty} C^2 dx = \int_{-\infty}^{+\infty} P^2 dx. \quad (4)$$

For a chosen value β , we determine the value for W and B as follows,

$$W = \frac{\sqrt{2}}{\sqrt{\pi\beta}}, \quad (5)$$

$$B = \frac{\sqrt{2\pi}}{4} W. \quad (6)$$

Apart from the profile, modelling should also account for the acoustic beam formation based on a plane wave expansion rooted in Fourier analysis. A harmonic sound field in space-time is described as,

$$\begin{aligned} \psi(x, z, t) &= \phi(x, z) e^{-i\omega t} \\ &= \sum_{n=-\frac{K-1}{2}}^{\frac{K-1}{2}} H_n e^{ik_{x,n}x + i\left(\sqrt{\frac{(2\pi f)^2}{c^2} - (k_{x,n})^2}\right)z} e^{-i\omega t}, \end{aligned} \quad (7)$$

in which H_n is the n -th Fourier coefficient obtained by FFT of $\phi(x, z)$ for the corresponding wave number $k_{x,n}$. Formula (7) takes into account the dispersion formula for non-viscous materials, to find $k_{z,n}$ for a frequency f , with,

$$(k_{x,n})^2 + (k_{z,n})^2 = \frac{(2\pi f)^2}{c^2}. \quad (8)$$

Beams are incident from water with a density of 1000 kg/m³ and sound velocity of 1480 m/s. As a representative example for experiments, we take a frequency of 1 MHz. The beams interact with a solid, having properties representative for brass, i.e. density 8100 kg/m³, ultrasound shear velocity 2270 m/s and ultrasound longitudinal velocity 4840 m/s.

Fourier analysis is done by FFT on 977 equidistant samples between -20 W and $+20$ W, although the results are plotted only in the interval -4 W to $+4$ W and sometimes $-W$ to $+W$ as seen subsequently.

The Fourier transform provides amplitude and phase of each plane wave that constitute the modelled bounded beam, having a wave vector $k_{x,n}$ along the profile axis (i.e. the surface of the transducer) determined by Fourier analysis, and a perpendicular wave vector component $k_{z,n}$, physically determined by the dispersion equation for harmonic plane waves, as applied in Formula (7) using

Formula (8). Adding up all the calculated plane waves as in Formula (7) results in the bounded beam.

Interaction of a bounded beam incident from water on an elastic solid is modeled by a linear combination of reflected plane waves each resulting from their respective incident plane waves in Formula (7), by assuming continuity of normal stress and normal displacement at the liquid–solid interface.

This study aims not to show Fourier analysis compared to competing methods such as a phase advance technique [40], but to reveal the influence of the beam shape.

3 Fourier effect

As explained in Section 2, we use the Fourier transform to model bounded beams. The advantage is that each involved plane wave is easily understood and allows one to simulate how it reflects off a solid by taking into account Snell's law and calculating the reflection and transmission coefficients for each wave. The technique may have one side effect of importance in the Schoch effect context, which we prefer to exclude by evidence before we continue, rather than ignore a priori.

3.1 The difference between backward radiation and backscattering

The model used in this work does not consider material inhomogeneities or surface roughness and does not consider any possible secondary diffraction effects on surface roughness caused by guided waves. Investigators like Nagy and Adler [41, 42] or Blakemore [43] provided experimental and theoretical explanations in which a clear distinction between backward radiation, caused by Fourier, as described in Section 3.2, and backscattering, caused by the material's inhomogeneities and surface roughness is important. Backscattering is, for instance, enhanced when sound is incident at the Rayleigh angle. Nagy and Adler [41, 42] clearly showed that this enhanced effect is solely caused by backscattering. In other words, minute backward radiation as in Section 3.2 exists, but is negligible compared to backscattering. Furthermore, the backscattering effect (not considered in the current work) produces an amplitude peak, in experiments, in the reflection of sound at the Rayleigh angle on the surface and is directed back to the transducer in a pulse-echo experiment.

3.2 Fourier effect causing backward radiation

The effect described here is not responsible for the observed high amplitude in the received echo at the Rayleigh angle during a pulse-echo experiment, as observed by Nagy and Adler [41, 42]. The Fourier effect is a minute backward radiation that exists for all angles of beam incidence and is caused by backward plane wave components in a bounded beam. For instance, if the incident beam is an outward spherical wave, the sound is always reflected to the transducer. If the beam is spherically focused, the

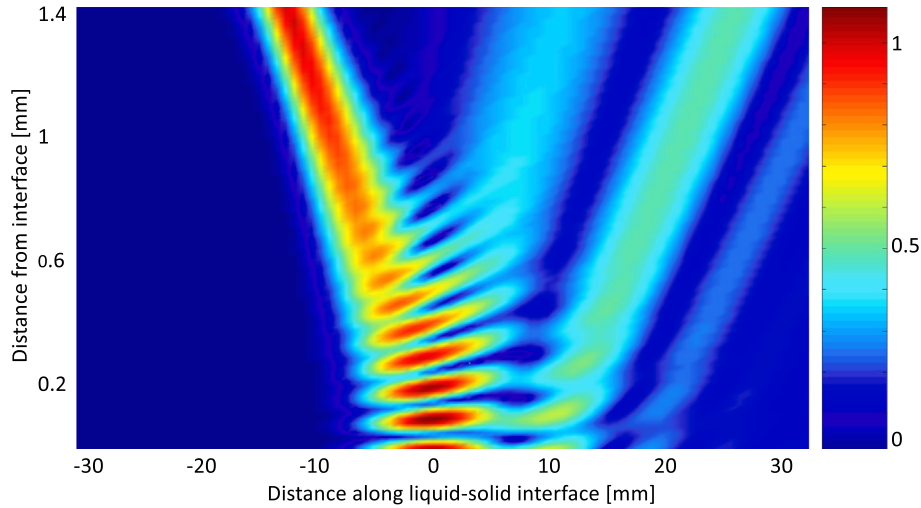


Figure 3. Numerical simulation of a bounded beam, incident from the top left of the figure, interacting with a thick brass sample, at the Rayleigh angle and for $W = 0.002$ m. The resulting sound field is caused by the interference of incident sound and reflected sound. The reflected sound consists of specular and non-specular sound, causing the Schoch effect.

sound will always be reflected, no matter what the angle of incidence. What these extremes have in common is a wide angular spectrum, consistent with the theoretical work of Norris [44] and others [45, 46]. A little fraction is reflected back for beams categorized between these two extremes.

Indeed, a proper Fourier-transform application to bounded beams requires, for symmetry reasons, the usage of forward and backward directed wave vector components along the transducer's surface, i.e. positive and negative values of $k_{x,n}$. In what follows, it is useful to keep an experiment as in Figure 2 in mind. For normal incident beams, half of the assumed plane wave bends towards the left-hand side and the other half towards the right-hand side. If such a beam is tilted to emit sound from top left to right bottom, more components will be directed to the right than the left. Still, some waves are directed to the left, and this means that, after reflection with a surface, basic geometry shows that a portion can bounce back to the emitter. The attribution of this small effect to a Fourier effect was made by Norris [47]. Important in the current study is that these waves may interfere with the other sound waves and influence the Schoch effect. However, the effect for regular beams is not as highly sensitive to whether the beam is incident at the Rayleigh angle as the backscattering effect studied by Nagy and Adler [41, 42]. When a beam has sharp edges, such as a *square beam*, backward radiation also prominently manifests because the sharp edge causes a wider angular spectrum than a smooth edge as with a *Gaussian beam*. For the same reason, a *square beam* will spread more prominently than a *Gaussian beam* while propagating, known as edge diffraction.

For demonstrative purposes, we apply the numerical technique described in Section 2, to 1 MHz sound incident from water on brass at the Rayleigh angle (44.04°). The material parameters are given in Section 2. The *Gaussian beam* width for this simulation is $W = 0.002$ m. The result is seen in Figure 3.

Separation of the sound field of Figure 3 to examine every possible contribution results in the observation that, for this case, the amplitude of the Fourier effect contribution to backward radiated sound is one millionth of the entire sound field. In other words, it shows, as Nagy and Adler [41, 42] have proven, that the Fourier effect is not the explanation for enhanced backscattering at the Rayleigh angle. Indeed, the Fourier effect is negligible, also in the discourse presented here.

4 Shape effect study

To compare the Schoch effect for different profiles, we present results for a small variety of β values, as defined in Formula (5), frequencies and for two distances: zero and W . Although the zero distance is classically used to study reflected beam profiles, it is physically problematic because a zero distance is only achieved for the center of the beam, while one portion has already emerged, and another is still emerging and therefore does not yet exist. A distance of W , or any other distance sufficiently far from the origin, is physically more straightforward because it ensures that the whole profile has emerged and propagates as reflected sound away from the solid. Note that the zero-distance result is usually smooth because propagation has not yet occurred, and hence there are no phase effects or edge diffraction observable caused by propagation. Both cases are essential to avoid false conclusions. Figures 4–10 expose the obtained results. The incident beam profile as in Formulas (1)–(3) are shown as colored dotted lines, serving as references and the reflected profiles in solid lines with the same colors. Only the Rayleigh angle results are shown because this is the only angle around which the Schoch effect is visible. From the shown results, it can be seen that the Schoch effect also appears from the other two profiles and is therefore not a typical Gauss effect. However, the

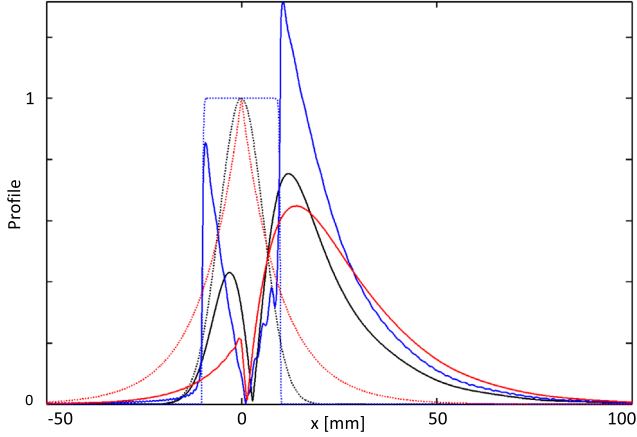


Figure 4. Absolute value of the incident (dotted lines) and reflected profiles (solid lines) at the Rayleigh angle, for 1 MHz, $\beta = 100/\text{m}$ after 0 m propagation distance. Comparison with Figure 7 shows that the smoothness of the result is caused by ignoring propagation.

extent and characteristics depend on the considered profile. Only for the first case do we present the real values to show the beam behavior in the real physical world. For the other cases, we only present absolute values as done traditionally.

4.1 Case study for 1 MHz and $\beta = 100/\text{m}$

Figure 4 shows results for the three profiles defined in the Formulas (1)–(3), at 1 MHz, for the sound incident at the Rayleigh angle, and characterized by $\beta = 100/\text{m}$, defined in Formula (5). Propagation after reflection is not taken into account, i.e., the profile represents the calculated profile at the origin, as is traditionally done. For the *Gaussian beam* (black curve), the reflected beam exposes the well-known Schoch effect, similar to that of Figure 2 or Figure 3 beautifully. When we look at the other profiles under the same circumstances a Schoch effect is still visible but different. The *Japanese temple beam* behaves similar to the *Gaussian beam*, likely due to Gaussian flanks' similarities with exponential flanks [48]. The *square beam* gives more striking results, however. Indeed, sharp peaks are seen, which, in an experiment, would show up as two narrow beams with a large zone in between consisting of low amplitude sound.

It is known that the null zone appearing with the Schoch effect is caused by destructive interference between specular and non-specular sound in the counter phase. This is seen in Figure 5 for each studied profile. Again, the *square beam* (blue) exposes the most spectacular results with significant separation between the specular lobe (left) and the non-specular one (right).

As mentioned, Figures 4 and 5 show reflected profiles at the origin. When sound is reflected on the surface and propagates away from the interface, phase influences can be expected and natural diffraction such as edge diffraction. The latter typically smoothens sharp profile peaks. Indeed, Figure 6 results in the same conclusions as to the previous

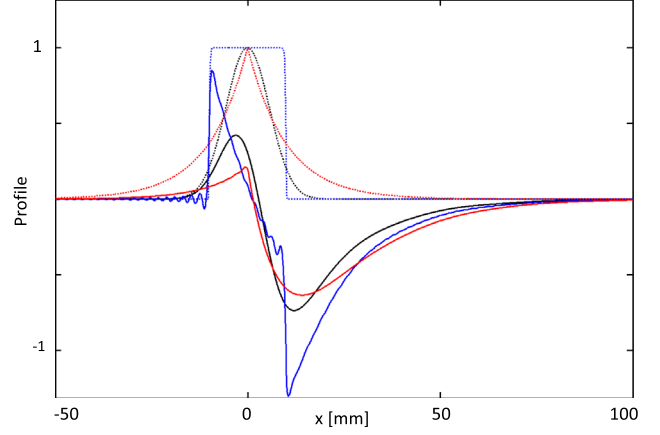


Figure 5. Real value of the incident (dotted lines) and reflected profiles (solid lines) at the Rayleigh angle, for 1 MHz, $\beta = 100/\text{m}$ after 0 m propagation distance. Propagation further influences these results, as shown in Figure 6.

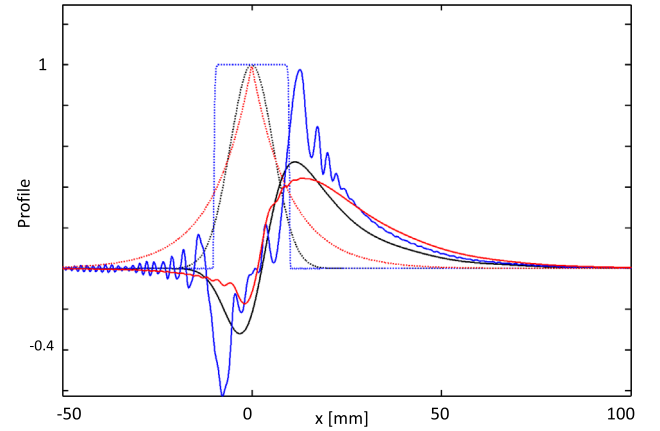


Figure 6. Real value of the incident (dotted lines) and reflected profiles (solid lines) at the Rayleigh angle, for 1 MHz, $\beta = 100/\text{m}$ after W propagation distance. The effect caused by propagation make the results rougher than the ones in Figure 5, where propagation is absent.

figure, except that some irregularities due to phase effects are visible. The sharp peaks of the reflected *square beam* (blue) are somewhat smoothed. Nevertheless, they are still relatively sharp, meaning that, in an actual experiment, the emerging narrow beams would only gently widen with distance.

For consistency with the explanation at zero distance in Figure 5, the real values are shown in Figure 7. The conclusions are still valid.

4.2 Case study for 1 MHz and $\beta = 600/\text{m}$

As mentioned earlier, we always compare beams based on the β value, ensuring all beams have the same energy. Whereas in the previous Section 4.1, values of 100 were taken, we now look at the results for $\beta = 600/\text{m}$ while using the same frequency. From a relative viewpoint, the beams are now much narrower than in the previous case. Still,

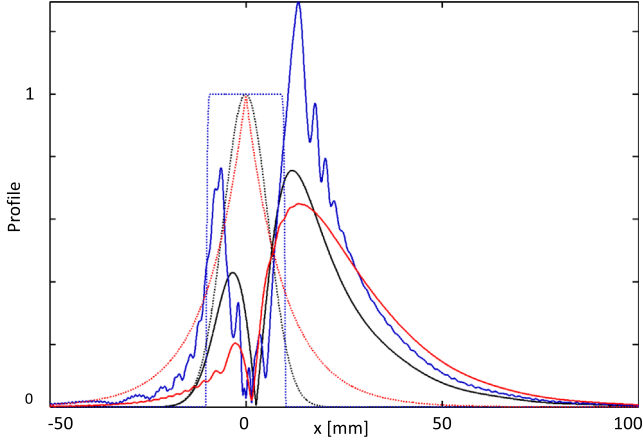


Figure 7. Absolute value of the incident (dotted lines) and reflected profiles (solid lines) at the Rayleigh angle, for 1 MHz, $\beta = 100/\text{m}$ after W propagation distance. Effects caused by propagation distinguish these results from the ones in Figure 4, where propagation is absent.

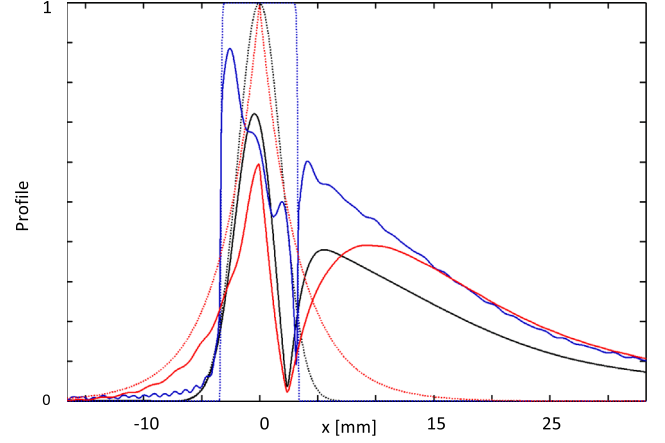


Figure 9. Absolute value of the incident (dotted lines) and reflected profiles (solid lines) at the Rayleigh angle, for 1 MHz, $\beta = 300/\text{m}$ after 0 m propagation distance. Comparison with Figures 4 and 8 reveals that the obtained results fall between those for $\beta = 100/\text{m}$ and $\beta = 600/\text{m}$.

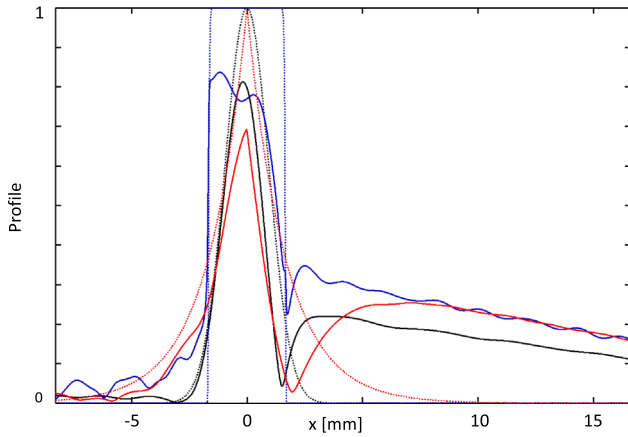


Figure 8. Absolute value of the incident (dotted lines) and reflected profiles (solid lines) at the Rayleigh angle, for 1 MHz, $\beta = 600/\text{m}$ after 0 m propagation distance. Comparison with Figure 4 shows that the higher value for β is mainly causing different effects for the non-Gaussian beams.

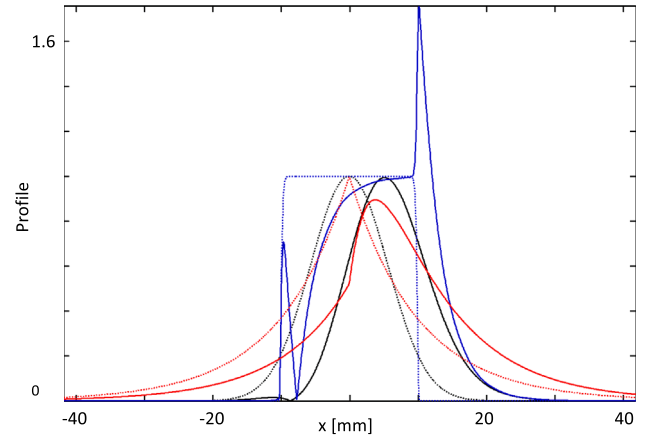


Figure 10. Absolute value of the incident (dotted lines) and reflected profiles (solid lines) at the Rayleigh angle, for 5 MHz, $\beta = 100/\text{m}$ after 0 m propagation distance. Comparison with Figure 4 reveals an influence on the Schoch effect, caused by the frequency, for each of the studied profiles.

the Schoch effect is visible for all beams, though different. There is now a weak and narrow null zone, while the non-specular beam is far-reaching. Every profile behaves somewhat similar, as shown in Figure 8.

4.3 Case study for 1 MHz and $\beta = 300/\text{m}$

Less extreme than the previous case is when β is in between the values in case A and case B. Here, we take $\beta = 300/\text{m}$ and the result is given in Figure 9. The conclusions are equally in between the two aforementioned extreme cases. Again, the *square shape* gives the most dramatic results, though less spectacular as in the first case. In other words, the wider the beam, the more striking the Schoch effect for *square beams*.

4.4 Case study for 5 MHz and $\beta = 100/\text{m}$

When the frequency is changed, the relative beamwidth is also different. If we repeat the calculations of cases A, B and C, for a higher frequency, then the narrow beams will behave as wider beams. We first have a look at case A, but now for a 5 MHz frequency, shown in Figure 10. As known from experience, the Schoch effect is hardly visible for the *Gaussian beam*. More interestingly is that the *square profile* exposes spectacular behavior again by exposing two sharp amplitude peaks left and right.

As in the previous cases, we expect that the observed sharp peaks occurring after reflection of the *square beam* smoothen with propagation. Figure 11 shows the result of Figure 10 after a propagation distance W . Indeed, the sharp peaks smoothen somewhat but are still there.

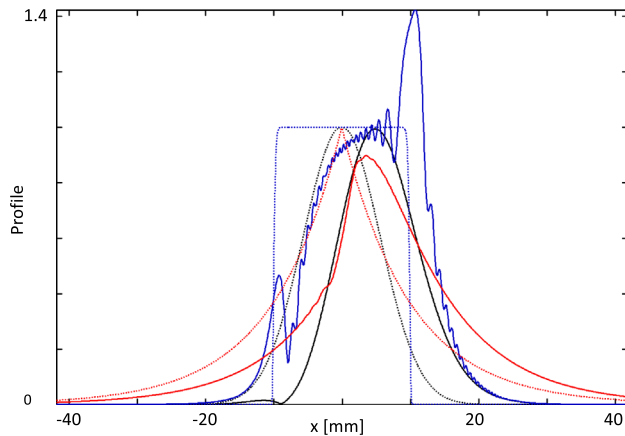


Figure 11. Absolute value of the incident (dotted lines) and reflected profiles (solid lines) at the Rayleigh angle, for 5 MHz, $\beta = 100/\text{m}$ after W propagation distance. As with the 1 MHz results of Figure 7, compared to Figure 4, influence caused by propagation is also observed in this case of 5 MHz.

5 Conclusions

Numerical simulations based on Fourier theory show that *square beams* exhibit a much stronger Schoch effect upon reflection from a solid than other, smoother profiles such as *Gaussian beams*. The study incorporates sound propagation to account for edge diffraction and reveals that the conclusions remain pertinent.

A suggestion for the further experimental study could be to investigate if wide *square beams* are also more efficient in Rayleigh wave generation than wide *Gaussian beams* because the Schoch effect physically accompanies Rayleigh wave generation. Additionally, the superior flexibility caused by today's progress in metamaterials for fabricating acoustic lenses may open new opportunities for experimentation with more exotic beam shapes than the ones reported in this work. An outspoken Schoch effect may contribute to the spatial filtering of sound beams or produce peculiar Rayleigh waves, possibly at a higher efficiency than traditional *Gaussian beams*.

Conflict of interest

The author declares that he has no conflicts of interest in relation to this article.

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