



Delamination thickness measurement based on Stoneley wave in bilayered composite structure

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Abstract – Introduction: For composite structures, delamination usually occurs at the interface. Ultrasonic guided waves have been widely used to detect the delamination. However, most researches focus on the measurement of the delamination length along the interface, and the measurement of the delamination thickness is less studied. **Method:** In this paper, based on the characteristic that the Stoneley wave only propagates along the interface, the reflection coefficient of interaction between Stoneley wave and delamination is used to measure the delamination thickness. The effects of delamination thickness and frequency on the reflection coefficient are investigated via dividing integral region and reciprocity theorem. Some experimental validations are carried out on two aluminum-steel bilayered composite plates with different delamination thickness. **Results:** It is found the reflection coefficient increases linearly at first, then its rate of increase slows down gradually, and finally becomes stable, in theory. And the experimental results can verify the theoretical relationship between the reflection coefficients and the delamination thickness. **Conclusion:** The variation of reflection coefficient provides a reference for the measurement of delamination thickness in Stoneley-wave-based non-destructive testing.

Keywords: Ultrasonic guided wave, Stoneley wave, Delamination, Reflection coefficient, Measurement

1 Introduction

Composite structures are widely used in the automotive and aerospace industry because of their excellent properties in recent years. Due to complex working conditions, there are various interfacial failure modes in composites structures, among which delamination is the most dominant and critical failure. It is necessary to inspect the structures by non-destructive testing (NDT) technology to ensure their integrity. For most large composite structures, the NDT based on ultrasonic guided waves is a quick, reliable method with only a few coupled transducers. Therefore, using the guided wave propagating along interface is an attractive solution to detect interfacial delamination in composite structures.

As early as the 19th century, people discovered and studied various types of ultrasonic guided waves. In 1885, a kind of wave that can propagate along the free surface of an elastic half space was found by Rayleigh [1]. Lamb introduced symmetric and asymmetric wave propagating stably in plates in 1917 [2]. It was reported that surface wave can propagate along the interface between two elastic half spaces for different materials by Stoneley in 1924 [3], so this kind of

wave called Stoneley wave. A considerable number of fundamental theory researches and corresponding experiments about ultrasonic guided waves have been made, and some scholars, such as Rose [4], collected these research results and recorded them in textbooks. To ultrasonic guided wave based non-destructive testing technology, using pulse reflecting to locate damages is the most direct application. For example, Ng and Veidt applied Lamb wave to inspect damage in composite laminates by analyzing the cross-correlation of reflected pulse envelope with excitation pulse envelope for each transducer pair and then superimposing results observed from each path [5]. Wang quantified the uncertainty on notch damage in a beam structure using Lamb wave responses through a Bayesian inference approach [6]. Golub et al. studied Lamb wave damage detection at the interface between a block and elastic plate by numerical analysis [7]. Kaur and Lata conducted a lot of important theoretical studies on guided wave propagation with memory-dependent derivatives (MDD), specifically, Rayleigh wave in translationally isotropic magneto thermoelastic homogeneous medium in the presence of mass diffusion and three phase lag heat transfer [8], Rayleigh Wave in isotropic magneto-thermoelastic diffusive medium [9] and Stoneley wave in translationally isotropic thermoelastic medium with two temperature and rotation [10].

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Once confirming the location of delamination damage, another essential work is to determine the size of delamination damage in one direction. The shape of delamination damages in composite structures can be accurately described with multi-dimensional measurement by guided wave method. For this purpose, scholars have done a lot of work. Toyama and Takatsubo detected interfacial delamination in composite laminates by Lamb wave and developed a specific experimental approach about measuring the delamination length with a simple wave-propagation modelling [11]. Mokhtari et al. introduced a new method of estimating damaged region via polygon reconstruction technique in tomography [12]. Singh et al. carried out numerical and experimental studies on mode conversion and scattering characteristics of fundamental anti-symmetric Lamb mode passing a semi-infinite delamination. Further, the reflection coefficients based on amplitude ratios have been computed to identify the defect [13]. Recently, Zima developed a novel Lamb wave propagation method, merely three sensors are demanded, for damage detection based on the reconstruction of the reflected wave-front shape [14]. Hervin et al. studied the influence of delamination shape and depth on guided wave scattering through a 3D finite element model and validated against the experiments [15].

In the above studies, some progress has been made in the field of Lamb wave based delamination damage detection technology. However, the Lamb wave has characteristics such as high-frequency dispersion, multi-mode characteristics, dispersing energy [4], which makes it not sensitive enough when used to detect interfacial damage. Unlike Lamb wave, the energy of Stoneley wave is concentrated at the propagating interface. Stoneley wave exhibited a clearly distinguished propagating velocity and low dispersion in the high-frequency range in Cui et al.'s work [16]. This means that the Stoneley wave has great advantages and broad application prospects in terms of detecting interface damage or evaluating interface binding state. Gardner et al. used ultrasonic guided waves to inspect delamination in interface between metal and some anisotropic material, and demonstrated the inspectability with both numerical and physical experimental results [17]. Ou et al. numerically simulated the Stoneley wave reflection by a horizontal fracture in a borehole by the variable grid spacing finite difference method, and reported the reflection coefficients affected by elastic modulus of medium [18]. Sahu et al. studied the propagation of Stoneley waves in interface between composed of rock and ice through a modeling method with the Coulomb frictional boundary conditions [19]. Anh et al. studied the propagation of Stoneley wave along the spring-contacted interface and his results showed that Stoneley wave is a good tool for evaluating the quality of imperfect bonds modeled as spring contacts [20].

Despite these studies, there are still two problems. One is that many researches are concentrated on the measurement of length of interfacial delamination in composite structures, but little on thickness. The other is that the Lamb wave-based testing will be limited by Lamb wave's

inherent limitations (high-frequency dispersion, multi-mode characteristics, dispersing energy). However, the Stoneley wave with energy concentrated at the interface can help solve those two problems. Therefore, this paper aims to analyze the reflection of Stoneley wave from a single interfacial delamination boundary at the two-material interface, and uses this two-dimensional plane simplified model to explain the concept of the interfacial delamination thickness measurement method. In terms of methods, we use the reflection coefficient to quantify the interaction between the Stoneley wave and the interfacial delamination, and obtain a specific function curve with the help of the reciprocity theorem. The front part of the curve reflects the linear relationship between the coefficient and the delamination thickness. Furthermore, the theoretical results are verified by experiment with slight deviation, which shows that it is feasible to detect the interfacial delamination thickness by Stoneley wave method. It will help the NDT applications of Stoneley wave and the reconstruction of damage at the interface.

2 Brief introduction of Stoneley wave

In this section, some brief introduction of Stoneley wave is presented. According to the potential functions and the boundary conditions, the secular equation of Stoneley wave are derived. Normalized displacement distribution and phase velocity are referenced in calculating the reflection coefficient.

There are two isotropic, elastic, homogeneous solid half-spaces Ω_1 ($z \geq 0$) and Ω_2 ($z \leq 0$) as shown in Figure 1. These two solid half-spaces bonded along $z = 0$ are defined by Lamé constant $\lambda_1, \mu_1, \lambda_2, \mu_2$ and density ρ_1, ρ_2 .

The displacement vector $\mathbf{u}^{(n)}$ in an elastic isotropic solid without body forces can be expressed via Helmholtz decomposition as the curl of the zero divergence vector and the gradient of a scalar [4]:

$$\begin{cases} \mathbf{u}^{(n)} = \nabla \phi_n + \nabla \times \boldsymbol{\psi}_n, \\ \nabla \cdot \boldsymbol{\psi}_n = 0 \end{cases}, \quad (1)$$

where $\nabla = \partial/\partial x \cdot \mathbf{i} + \partial/\partial z \cdot \mathbf{k}$, $\mathbf{u}^{(n)} = (u_x^{(n)}, u_z^{(n)})$, n is the number in Ω_1 or Ω_2 , ϕ_n are scalar potentials and $\boldsymbol{\psi}_n$ are vector potentials. For general plane strain problems, there are some constraints that $\mathbf{w}_{n,x} = \mathbf{w}_{n,z} \equiv 0$, $\mathbf{w}_{n,y} = \mathbf{w}_{n,y}(x, z)$ and $\phi_n = \phi_n(x, z)$. For simplicity potential $\boldsymbol{\psi}_{n,y}$ function is hereafter represented by ψ_n .

The expressions of plane harmonic wave corresponding in Ω_1 and Ω_2 can be derived in the form [4]:

For Ω_1

$$\begin{cases} \phi_1 = A_1 e^{\alpha_1 z} e^{i(kx - \omega t)} \\ \psi_1 = B_1 e^{\beta_1 z} e^{i(kx - \omega t)} \end{cases}. \quad (2)$$

For Ω_2

$$\begin{cases} \phi_2 = A_2 e^{-\alpha_2 z} e^{i(kx - \omega t)} \\ \psi_2 = B_2 e^{-\beta_2 z} e^{i(kx - \omega t)} \end{cases} \quad (3)$$

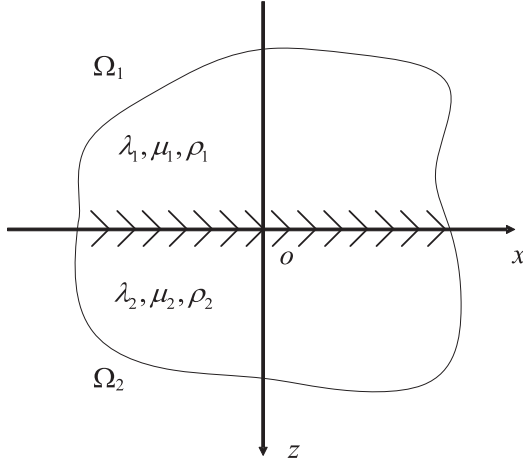


Figure 1. Bonded interface with different materials.

k is wave number, ω is angular frequency. The solution of wave equation accurately includes $e^{i(kx \pm \omega t)}$ two items, where the positive sign indicates that as time t increases, the position coordinate x also increases, while the negative sign indicates the opposite. So $kx - \omega t$ represents the positive direction of the traveling wave along the x -axis, $kx + \omega t$ represents the traveling wave in the negative x -axis direction. We only consider traveling wave propagating in the positive x -axis direction here, so $kx - \omega t$ is taken. Other coefficients are defined as follow:

$$\alpha_n^2 = \frac{\omega^2}{c^2} - \frac{\omega^2}{(c_L^{(n)})^2}, \beta_n^2 = \frac{\omega^2}{c^2} - \frac{\omega^2}{(c_T^{(n)})^2}, k = \frac{\omega}{c} \quad (4)$$

c is the Stoneley wave phase velocity. $c_T^{(n)} = \sqrt{(\lambda_n + 2\mu_n)/\rho_n}$ is the longitudinal wave velocities. $c_T^{(n)} = \sqrt{\mu_n/\rho_n}$ is the shear wave velocities.

According to assumption of plane strain, we can calculate the displacement and stress in terms of the potentials [4]:

$$\begin{aligned} u_x^{(n)} &= \frac{\partial \phi_n}{\partial x} + \frac{\partial \psi_n}{\partial z}; u_z^{(n)} = \frac{\partial \phi_n}{\partial z} + \frac{\partial \psi_n}{\partial x}; \\ \tau_{xz}^{(n)} &= \mu_n \left(\frac{\partial u_z^{(n)}}{\partial x} + \frac{\partial u_x^{(n)}}{\partial z} \right); \tau_{xx}^{(n)} = \lambda_n \left(\frac{\partial u_x^{(n)}}{\partial x} + \frac{\partial u_z^{(n)}}{\partial z} \right) \\ &+ 2\mu_n \frac{\partial u_x^{(n)}}{\partial x}; \tau_{zz}^{(n)} = \lambda_n \left(\frac{\partial u_x^{(n)}}{\partial x} + \frac{\partial u_z^{(n)}}{\partial z} \right) + 2\mu_n \frac{\partial u_z^{(n)}}{\partial z}. \end{aligned} \quad (5)$$

Because the two solids are perfectly bonded $z=0$, the boundary conditions are listed as follow:

$$\begin{aligned} u_x^{(1)} &= u_x^{(2)}; & u_z^{(1)} &= u_z^{(2)} \\ \tau_{xz}^{(1)} &= \tau_{xz}^{(2)}; & \tau_{zz}^{(1)} &= \tau_{zz}^{(2)}. \end{aligned} \quad (6)$$

Substitute equations (2), (3), and (5) into equation (6). The following secular equation can be obtained:

See the Equation (7) bottom of the page

To guarantee equation (7) is always equal to zero, the determinant of matrix $\mathbf{A}$ must be zero. It creates a univariate equation with c as only variable. Coincidentally, the circular frequency disappears in equation (7), so the Stoneley wave is non-dispersive. Besides, the univariate equation has only one solution, which means there is only one mode of Stoneley wave. The properties above make the Stoneley wave more suitable for non-destructive testing.

Equation (7) is a linear homogeneous equation, the uncertainty coefficients: A_1 , B_1 , A_2 and B_2 are linearly dependent. Therefore, one of uncertainty coefficient A_1 , B_1 , A_2 and B_2 can be used to represent others. For example, B_1 , A_2 and B_2 can be expressed by A_{1as} :

$$\begin{bmatrix} B_1 \\ A_2 \\ B_2 \end{bmatrix} = -A_1 \begin{bmatrix} \mathbf{A}(2,2) & \cdots & \mathbf{A}(2,4) \\ \vdots & \ddots & \vdots \\ \mathbf{A}(4,2) & \cdots & \mathbf{A}(4,4) \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{A}(2,1) \\ \mathbf{A}(3,1) \\ \mathbf{A}(4,1) \end{bmatrix}. \quad (8)$$

The expressions of displacement and stress are obtained via only one parameter A_1 as:

$$\begin{cases} u_x^{(n)}(x, z, \omega) = A_1 U_x^{S+, (n)}(z, \omega) e^{i(kx - \omega t)} \\ u_z^{(n)}(x, z, \omega) = A_1 U_z^{S+, (n)}(z, \omega) e^{i(kx - \omega t)} \\ \tau_{xx}^{(n)}(x, z, \omega) = A_1 U_{xx}^{S+, (n)}(z, \omega) e^{i(kx - \omega t)} \\ \tau_{xz}^{(n)}(x, z, \omega) = A_1 U_{xz}^{S+, (n)}(z, \omega) e^{i(kx - \omega t)} \\ \tau_{zz}^{(n)}(x, z, \omega) = A_1 U_{zz}^{S+, (n)}(z, \omega) e^{i(kx - \omega t)} \end{cases} \quad (9)$$

The function $U(z, \omega)$ and $T(z, \omega)$ are obtained by substituting a solution of the equation back into equation (5), and the form is not unique. The classical form can be referred to in reference [21]. The superscript $S+$ means Stoneley wave propagating forward to x -axis. If the expressions of displacement and stress of Stoneley wave propagating along the negative direction of the x -axis needs to be obtained, the value of phase velocity c in equation (8) should be negative. In this case, Stoneley wave propagating backward to x -axis can be donated as $S-$.

The introduction of Stoneley wave and the expressions of the displacement and stress of Stoneley wave lay a foundation for calculating the reflection coefficient.

$$\mathbf{A} \begin{bmatrix} A_1 & B_1 & A_2 & B_2 \end{bmatrix}^T = 0$$

$$\mathbf{A} = \begin{bmatrix} ik & \beta_1 & -ik & \beta_2 \\ \alpha_1 & -ik & \alpha_2 & ik \\ 2i\alpha_1\mu_1k & \mu_1(\beta_1^2 + k^2) & 2i\alpha_2\mu_2k & -\mu_2(\beta_2^2 + k^2) \\ \lambda_1(\alpha_1^2 - k^2) + 2\alpha_1^2\mu_1 & -2i\beta_1\mu_1k & -\lambda_2(\alpha_2^2 - k^2) - 2\alpha_2^2\mu_2 & -2i\beta_2\mu_2k \end{bmatrix}. \quad (7)$$

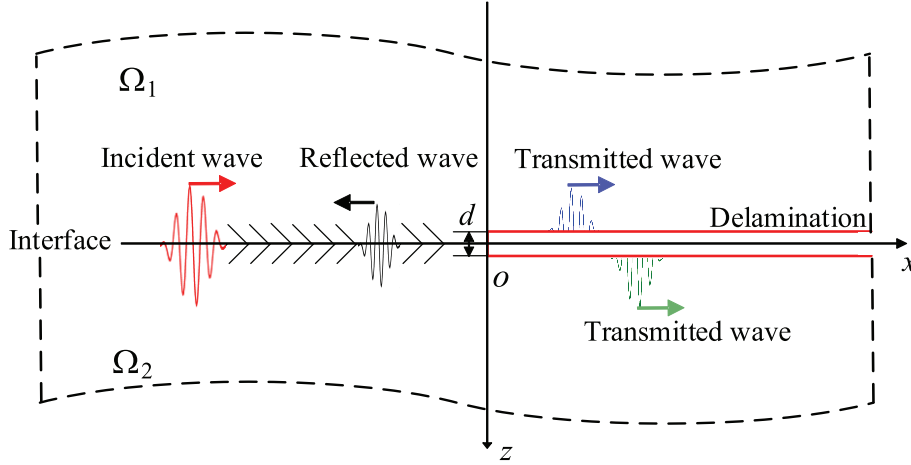


Figure 2. Diagram of reflection and transmission.

3 Reflection coefficient calculation

In this part, the interaction between Stoneley wave and delamination is computed. The variation of reflection coefficient with delamination thickness and frequency is calculated. The influence of Stoneley wave energy distribution on the reflection coefficient is discussed. The relationship between reflection coefficient and delamination thickness provides theoretical support for measuring delamination thickness via reflection coefficient.

3.1 Problem modeling

Consider two half-spaces Ω_1 and Ω_2 as in the previous section, $z > 0$ and $z \leq 0$, respectively, relative to a Cartesian coordinate system, (x, y, z) . The solids Ω_1 and Ω_2 are bonded perfectly together along $z = 0$ in $x < 0$. In space $x \geq 0$, there is a delamination with a thickness of d in the z direction and a rectangular shape symmetric along the x -axis. As shown in Figure 2, a Stoneley wave propagates forward along the x -axis and meets the delamination. In order to emphasize the influence of delamination thickness on reflection coefficient and restrain the influence of delamination length on reflection coefficient. The delamination length along x -axis is set to be much larger than the wavelength of incident wave. There are reflected and transmitted wave generated by the interaction between Stoneley wave and delamination. For the reflected wave, the boundary condition of perfectly bonded interface is unchanged, so the reflected wave is still a Stoneley wave. However, for the transmitted wave, the boundary condition of perfectly bonded interface is changed to free surfaces. Therefore, the transmitted wave should convert to two Rayleigh waves. We will focus on the reflected Stoneley wave in what follows.

On the whole, the elastodynamic reciprocity theorem provides a relationship between displacements, stress components and body forces for two different states of the same body [22]. For the two-dimensional multilayered case of time-harmonic quantities, the relation is of the form [22]:

$$\sum_{n=1}^{n=N} \int_{V^{(n)}} \left(f_j^{A,(n)} u_j^{B,(n)} - f_j^{B,(n)} u_j^{A,(n)} \right) dV^{(n)} = \sum_{n=1}^{n=N} \int_{S^{(n)}} \left(\tau_{ij}^{B,(n)} u_j^{A,(n)} - \tau_{ij}^{A,(n)} u_j^{B,(n)} \right) n_i dS^{(n)} \quad (10)$$

where $S^{(n)}$ defines a contour around an area defined by $V^{(n)}$ without interfaces, and $f_j^{(n)}$, $u_j^{(n)}$ and $\tau_{ij}^{(n)}$ denote the components of forces, displacements and stresses in the n th material, respectively. n_i is normal vectors of $S^{(n)}$. Subscript i and j indicate x direction and z direction. Here, the superscripts A and B denote two elastodynamic states. For a traction-free system, the left side of equation (10) is zero, equation (10) transfers as follow:

$$\int_{S^{(n)}} \left(\tau_{ij}^{B,(n)} u_j^{A,(n)} - \tau_{ij}^{A,(n)} u_j^{B,(n)} \right) n_i dS^{(n)} = 0. \quad (11)$$

Since the material properties of the two solids Ω_1 and Ω_2 are not the same, the phase velocity of the two Rayleigh waves in Ω_1 and Ω_2 are also different. Therefore, the integral region in Ω_1 and Ω_2 are analyzed independently. Hereafter, taking solid Ω_1 as an example, the selection of integral region and integral path is introduced in detail. According to the propagation characteristics of Stoneley wave and Rayleigh wave, the integral region and integral path in Ω_1 are selected as Figure 3.

In Figure 3, the path S are divided into six parts indicating $S = S_1 + S_2 + S_3 + S_4 + S_5 + S_6$. S_1 overlaps with the interface, S_2 overlaps with delamination thickness boundary along the z -direction, and S_3 overlaps with delamination length boundary along the x -direction. S_4 and S_6 set at $x = a$ and $x = b$ are straight and perpendicular to x -axis. In this case, we choose the integral contour S in the counter-clockwise direction. Note that the coordinates a , b are selected large enough for body wave to be neglected in reciprocity theorem. In addition, a has to meet other requirements, which will be explained later. To simplify calculation, the path S_5 are set at infinity. Because the Stoneley wave and Rayleigh wave energy approach zero at infinity, the integral of S_5 can vanish.

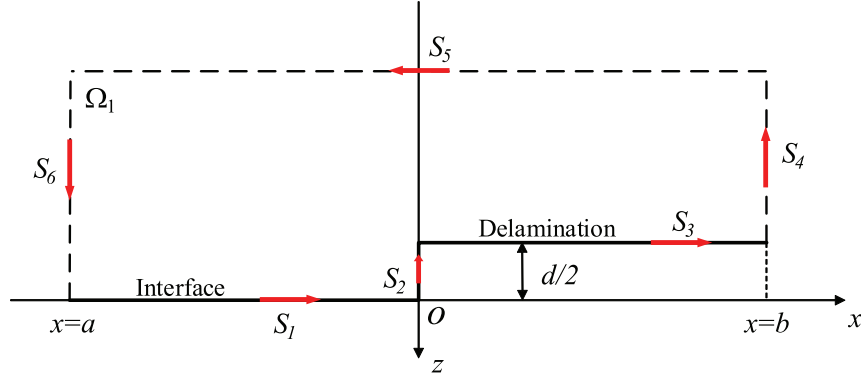


Figure 3. Integral region and integral path of reciprocity theorem in Ω_1 .

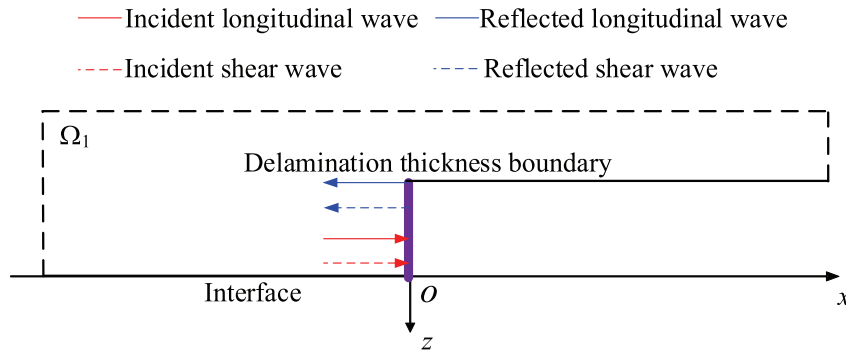


Figure 4. Interaction of incident Stoneley wave with delamination thickness boundary.

According to equation (11) and Figure 3, in order to calculate reflection coefficient, the displacement and stress components along the integral contour S should be confirmed. The displacement and stress components along the integral contour S_1 and S_6 are caused by Stoneley wave. The displacement and stress components along the integral path S_3 and S_4 are caused by Rayleigh wave. Therefore, the integral along S_1 , S_3 , S_4 and S_6 are relatively easy to calculate. In comparison, due to the interaction between the Stoneley wave and the delamination, the displacements and stresses along S_2 are difficult to be obtained. In next part, the method of calculating the displacement and stress components along the delamination thickness boundary (S_2) by Snell's law is mainly introduced.

3.2 Analysis on the delamination thickness boundary

We analyse the interaction of Stoneley wave with delamination and calculate displacement and stress components on thickness boundary in this part.

Stoneley wave is formed by coupling longitudinal wave and shear wave. As the Stoneley wave encounters the delamination in Ω_1 , the incident longitudinal wave and shear wave are reflected by delamination thickness boundary, converting the reflected longitudinal wave and shear wave, as shown in Figure 4. In Ω_1 , the potential functions which omit the harmonic term around reflection boundary

of the longitudinal and shear wave of the incident and reflected wave are expressed as equations (12) and (13):

$$\begin{cases} \phi_1^{in} = A_1^{in} e^{\alpha_1 z} e^{ikx} \\ \psi_1^{in} = B_1^{in} e^{\beta_1 z} e^{ikx}, \end{cases} \quad (12)$$

$$\begin{cases} \phi_1^{re} = A_1^{re} e^{ik_L^{(1)} x} \\ \psi_1^{re} = B_1^{re} e^{ik_T^{(1)} x}. \end{cases} \quad (13)$$

The four undetermined coefficients: A_1^{in} , B_1^{in} , A_1^{re} , and B_1^{re} represent the amplitudes of incident longitudinal wave, incident shear wave, reflected longitudinal wave and reflected shear wave, respectively. The expressions equations (12) and (13) of the potential function of the incident and reflected wave are not the same. Because the longitudinal and shear waves of incident wave have completed the coupling and formed the Stoneley wave. Conversely, the reflected longitudinal and shear wave have only undergone the reflection process around delamination thickness boundary, and not completed the coupling process. Therefore, the reflected wave has not converted to Stoneley wave around delamination thickness boundary, and it is still an ordinary form of longitudinal wave and shear wave, as shown in Figure 4. The potential functions of longitudinal and shear wave around delamination thickness boundary at $(0, z_0)$ are obtained as below:

$$\begin{cases} \phi_1^{(0,z_0)} = \phi_1^{in} + \phi_1^{re} \\ \psi_1^{(0,z_0)} = \psi_1^{in} + \psi_1^{re} \end{cases} \quad (14)$$

Using the relationship of equation (5) among potential functions $(\phi_1^{(0,z_0)}, \psi_1^{(0,z_0)})$, displacements $(u_x^{(1),(0,z_0)}, u_z^{(1),(0,z_0)})$ and stresses $(\tau_{xz}^{(1),(0,z_0)}, \tau_{zz}^{(1),(0,z_0)}, \tau_{xx}^{(1),(0,z_0)})$, the displacement and stress components at $(0, z_0)$ with four undetermined coefficient: $A_1^{in}, B_1^{in}, A_1^{re}, B_1^{re}$ can be obtained. The tangential and normal stresses along the delamination thickness boundary should be zero owing to the physical characteristics of the boundary. So, the stress components $\tau_{xz}^{(1),(0,z_0)}$ and $\tau_{xx}^{(1),(0,z_0)}$ at $(0, z_0)$ satisfy the following relationship:

$$\begin{cases} \tau_{xx}^{(1),(0,z_0)} = 0 \\ \tau_{xz}^{(1),(0,z_0)} = 0 \end{cases} \quad (15)$$

In equations (12) and (13), there are four undetermined coefficients: $A_1^{in}, B_1^{in}, A_1^{re}, B_1^{re}$. Only one of the four undetermined coefficients is independent, and the other three undetermined coefficients can be represented by it. In Eq. (9), since the incident wave is Stoneley wave, B_1^{in} can be represented by A_1^{in} . It means A_1^{re} and B_1^{re} can be expressed by A_1^{in} and B_1^{in} according to equation (15). The displacement and stress components at any point $(0, z_0)$ around the delamination thickness boundary in Figure 4 can be represented by A_1^{in} as following expressions:

$$\begin{cases} u_x^{de,(1)}(0, z_0, \omega) = A_1^{in} U_x^{de,(1)}(0, z_0, \omega) \\ u_z^{de,(1)}(0, z_0, \omega) = A_1^{in} U_z^{de,(1)}(0, z_0, \omega) \\ \tau_{xx}^{de,(1)}(0, z_0, \omega) = 0 \\ \tau_{xz}^{de,(1)}(0, z_0, \omega) = 0 \\ \tau_{zz}^{de,(1)}(0, z_0, \omega) = A_1^{in} T_{zz}^{de,(1)}(0, z_0, \omega) \end{cases} \quad (16)$$

According to equation (16), the displacement and stress components around the delamination thickness boundary can be obtained. These component expressions form the basis for calculations in the following parts.

3.3 Reflection coefficient

The main work is calculating the reflection coefficient based on the reciprocity theorem in this part. As shown in Figure 2, one reflected and two transmitted waves are generated due to the interaction of Stoneley wave with the delamination. Considering the change of boundary conditions, the transmitted wave convert to Rayleigh wave. The reflected wave returns back its original path, so it is still a Stoneley wave. According to equation (9), the corresponding expressions of the reflected Stoneley wave is equation (17) and of the transmitted Rayleigh wave is equation (18):

$$\begin{cases} u_x^{S-, (n)}(x, z, \omega) = A_1^{re} U_x^{S-, (1)}(z, \omega) e^{i(-kx - \omega t)} \\ u_z^{S-, (n)}(x, z, \omega) = A_1^{re} U_z^{S-, (1)}(z, \omega) e^{i(-kx - \omega t)} \\ \tau_{xx}^{S-, (n)}(x, z, \omega) = A_1^{re} U_{xx}^{S-, (1)}(z, \omega) e^{i(-kx - \omega t)} \\ \tau_{xz}^{S-, (n)}(x, z, \omega) = A_1^{re} U_{xz}^{S-, (1)}(z, \omega) e^{i(-kx - \omega t)} \\ \tau_{zz}^{S-, (n)}(x, z, \omega) = A_1^{re} U_{zz}^{S-, (1)}(z, \omega) e^{i(-kx - \omega t)} \end{cases} \quad (17)$$

$$\begin{cases} u_x^{R+, (1)}(x, z, \omega) = C_1^{tr} U_x^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \\ u_z^{R+, (1)}(x, z, \omega) = C_1^{tr} U_z^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \\ \tau_{xx}^{R+, (1)}(x, z, \omega) = C_1^{tr} U_{xx}^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \\ \tau_{xz}^{R+, (1)}(x, z, \omega) = C_1^{tr} U_{xz}^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \\ \tau_{zz}^{R+, (1)}(x, z, \omega) = C_1^{tr} U_{zz}^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \end{cases} \quad (18)$$

where:

$$k_1 = \frac{\omega}{c_1} \quad (19)$$

It should be noted that, in equation (18), the derivation of the expression of Rayleigh wave has been introduced in detail in reference [23], so it is not presented in the paper. The specific derivation of expressions of Rayleigh wave can be referred to reference [23].

Since, the reflected Stoneley wave propagates along the negative direction of the x -axis, the phase velocity and wave number should be negative and its expression is denoted as S^- . c_1 is the phase velocity of Rayleigh wave in Ω_1 . Undetermined coefficients: A_1^{re} and C_1^{tr} represent the amplitudes of reflected and transmitted wave.

According to equation (11), it is necessary to confirm the two mentioned elastodynamic states. In different elastodynamic states, the displacement and stress components caused by different types of guided waves along the integral path also need to be determined. State A is selected as the actual state. In state A , the displacement and stress components along S_1 and S_6 are caused by reflected Stoneley wave. The displacement and stress components along S_3 and S_4 are caused by transmitted Rayleigh wave. The displacement and stress components along S_2 are caused by the interaction between incident Stoneley wave and delamination thickness boundary which has been discussed in Section 3.2. State B is selected as the virtual state. The virtual state B is assumed as the Rayleigh wave propagating along the contour S_1 and S_3 in Ω_1 forward x -axis. Therefore, the displacement and stress components on all the integral path in state B are caused by Rayleigh wave. The expression of Rayleigh wave in the state B is as follow:

$$\begin{cases} u_x^{R+, (1)}(x, z, \omega) = E_1 U_x^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \\ u_z^{R+, (1)}(x, z, \omega) = E_1 U_z^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \\ \tau_{xx}^{R+, (1)}(x, z, \omega) = E_1 T_{xx}^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \\ \tau_{xz}^{R+, (1)}(x, z, \omega) = E_1 T_{xz}^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \\ \tau_{zz}^{R+, (1)}(x, z, \omega) = E_1 T_{zz}^{R+, (1)}(z, \omega) e^{i(k_1x - \omega t)} \end{cases} \quad (20)$$

Undetermined coefficient E_1 represents the amplitude of Rayleigh wave in virtual state and is not zero. Substitute the integral path and the two states into equation (11) and get the following expression:

See the Equation (21) bottom of next page

Assembling equations (17) and (20), the integral on S_1 can be calculated as:

See the Equation (22) bottom of next page

In equation (22), since the integral area can be selected, we can set: $(k_1 - k)a = 2m\pi$, m is a positive integer. When an appropriate integral area is selected: $a = 2m\pi/(k_1 - k)$, then the integral on S_1 is zero.

Assembling equations (16) and (20) and note the boundary condition on S_2 : $\tau_{xz}^{de,(1)} = \tau_{xx}^{de,(1)} = 0$, the integral on S_2 can be calculated as:

See the Equation (23) bottom of the page

Assembling equations (18) and (20), the integral on S_3 can be calculated as:

$$C_1^{tr} E_1 \int_0^b (T_{xz}^{R+, (1)} U_x^{R+, (1)} e^{2ik_1 x} - T_{xz}^{R+, (1)} U_x^{R+, (1)} e^{2ik_1 x} + T_{zz}^{R+, (1)} U_z^{R+, (1)} e^{2ik_1 x} - T_{zz}^{R+, (1)} U_z^{R+, (1)} e^{2ik_1 x})|_{z=-d/2} dx = 0. \quad (24)$$

Assembling equations (18) and (20), the integral on S_4 can be calculated as:

$$C_1^{tr} E_1 \int_{-d/2}^{-\infty} (T_{xx}^{R+, (1)} U_x^{R+, (1)} e^{2ik_1 x} - T_{xx}^{R+, (1)} U_x^{R+, (1)} e^{2ik_1 x} + T_{xz}^{R+, (1)} U_z^{R+, (1)} e^{2ik_1 x} - T_{xz}^{R+, (1)} U_z^{R+, (1)} e^{2ik_1 x})|_{x=b} dz = 0. \quad (25)$$

In equations (24) and (25), since the appropriate virtual state is selected, the integrals on contour S_3 and S_4 are both zero.

Since the integral path S_5 is set at infinity, and the displacement and stress components of Stoneley wave and Rayleigh wave approach zero at infinity. So the integral is also zero along S_5 .

Assembling equations (17) and (20) and note: $a = 2m\pi/(k_1 - k)$, the integral on S_6 can be calculated as:

See the Equation (26) bottom of the page

Assembling equations (23) and (26), equation (21) can be simplified as:

See the Equation (27) bottom of the page

Coincidentally, the undetermined coefficient E_1 representing the magnitude of the virtual state is reduced. Therefore, we can obtain:

See the Equation (28) top of next page

Using similar analysis and calculation methods, we can get similar results in Ω_2 as equation (29). It should be noted

$$\begin{aligned} & \int_{S_1+S_3+S_5} (\tau_{xz}^{B,(1)} u_x^{A,(1)} - \tau_{xz}^{A,(1)} u_x^{B,(1)} + \tau_{zz}^{B,(1)} u_z^{A,(1)} - \tau_{zz}^{A,(1)} u_z^{B,(1)}) dx + \\ & \int_{S_2+S_4+S_6} (\tau_{xx}^{B,(1)} u_x^{A,(1)} - \tau_{xx}^{A,(1)} u_x^{B,(1)} + \tau_{xz}^{B,(1)} u_z^{A,(1)} - \tau_{xz}^{A,(1)} u_z^{B,(1)}) dz = 0. \end{aligned} \quad (21)$$

$$\begin{aligned} & A_1^{re} E_1 \int_a^0 (T_{xz}^{R+, (1)} U_x^{S-, (1)} e^{i(k_1-k)x} - T_{xz}^{S-, (1)} U_x^{R+, (1)} e^{i(k_1-k)x} + T_{zz}^{R+, (1)} U_z^{S-, (1)} e^{i(k_1-k)x} - T_{zz}^{S-, (1)} U_z^{R+, (1)} e^{i(k_1-k)x})|_{z=0} dx \\ & = \frac{A_1^{re} E_1 (T_{xz}^{R+, (1)} U_x^{S-, (1)} - T_{xz}^{S-, (1)} U_x^{R+, (1)} + T_{zz}^{R+, (1)} U_z^{S-, (1)} - T_{zz}^{S-, (1)} U_z^{R+, (1)}) (1 - e^{i(k_1-k)a})}{i(k_1 - k)}. \end{aligned} \quad (22)$$

$$\begin{aligned} & E_1 \int_0^{-d/2} (T_{xx}^{R+, (1)} u_x^{de, (1)} - \tau_{xx}^{de, (1)} U_x^{R+, (1)} + T_{xz}^{R+, (1)} u_z^{de, (1)} - \tau_{xz}^{de, (1)} U_z^{R+, (1)}) e^{ik_1 x}|_{x=0} dz \\ & = A_1^{in} E_1 \int_0^{-d/2} (T_{xx}^{R+, (1)} U_x^{de, (1)} + T_{xz}^{R+, (1)} U_z^{de, (1)}) e^{ik_1 x}|_{x=0} dz. \end{aligned} \quad (23)$$

$$\begin{aligned} & A_1^{re} E_1 \int_{-\infty}^0 (T_{xx}^{R+, (1)} U_x^{S-, (1)} e^{i(k_1-k)x} - T_{xx}^{S-, (1)} U_x^{R+, (1)} e^{i(k_1-k)x} + T_{xz}^{R+, (1)} U_z^{S-, (1)} e^{i(k_1-k)x} - T_{xz}^{S-, (1)} U_z^{R+, (1)} e^{i(k_1-k)x})|_{x=a} dz \\ & = A_1^{re} E_1 e^{i(k_1-k)a} \int_{-\infty}^0 (T_{xx}^{R+, (1)} U_x^{S-, (1)} - T_{xx}^{S-, (1)} U_x^{R+, (1)} + T_{xz}^{R+, (1)} U_z^{S-, (1)} - T_{xz}^{S-, (1)} U_z^{R+, (1)}) dz \\ & = A_1^{re} E_1 \int_{-\infty}^0 (T_{xx}^{R+, (1)} U_x^{S-, (1)} - T_{xx}^{S-, (1)} U_x^{R+, (1)} + T_{xz}^{R+, (1)} U_z^{S-, (1)} - T_{xz}^{S-, (1)} U_z^{R+, (1)}) dz. \end{aligned} \quad (26)$$

$$\begin{aligned} & A_1^{re} E_1 \int_{-\infty}^0 (T_{xx}^{R+, (1)} U_x^{S-, (1)} - T_{xx}^{S-, (1)} U_x^{R+, (1)} + T_{xz}^{R+, (1)} U_z^{S-, (1)} - T_{xz}^{S-, (1)} U_z^{R+, (1)}) dz \\ & = A_1^{in} E_1 \int_{-\infty}^{-d/2} (T_{xx}^{R+, (1)} U_x^{de, (1)} + T_{xz}^{R+, (1)} U_z^{de, (1)}) e^{ik_1 x}|_0 dz = 0. \end{aligned} \quad (27)$$

$$\frac{A_1^{re}}{A_1^{in}} = \frac{-\int_0^{-d/2} (T_{xx}^{R+, (1)} U_x^{de, (1)} + T_{xz}^{R+, (1)} U_z^{de, (1)}) dz}{\int_{-\infty}^0 (T_{xx}^{R+, (1)} U_x^{S-, (1)} - T_{xx}^{S-, (1)} U_x^{R+, (1)} + T_{xz}^{R+, (1)} U_z^{S-, (1)} - T_{xz}^{S-, (1)} U_z^{R+, (1)}) dz}. \quad (28)$$

$$\frac{A_2^{re}}{A_2^{in}} = \frac{-\int_{d/2}^0 (T_{xx}^{R+, (2)} U_x^{de, (2)} + T_{xz}^{R+, (2)} U_z^{de, (2)}) dz}{\int_0^{+\infty} (T_{xx}^{R+, (2)} U_x^{S-, (2)} - T_{xx}^{S-, (2)} U_x^{R+, (2)} + T_{xz}^{R+, (2)} U_z^{S-, (2)} - T_{xz}^{S-, (2)} U_z^{R+, (2)}) dz}. \quad (29)$$

that the direction of integral in Ω_2 needs to be consistent with the direction of integral in Ω_1 .

See the Equation (29) top of the page

Since during the analysis of reflection, we pay more attention to the variation of amplitude. Thus the reflection coefficient can be defined as:

$$R = \left| \frac{A_1^{re}}{A_1^{in}} + \frac{A_2^{re}}{A_2^{in}} \right|. \quad (30)$$

In equations (29) and (30), the upper and lower limits of the numerator integrals include the delamination thickness and the expressions of displacement and stress include the frequency. When the frequency is unchanged, the integral of numerator is mainly affected by delamination thickness. When the delamination thickness is unchanged, the integral of numerator and denominator are both affected by frequency. The change of frequency can result in the change of energy distribution of Stoneley wave. Therefore, equations (29) and (30) present the influence of delamination thickness and Stoneley wave properties on reflection coefficient. In the next section, the effects of delamination thickness and excitation frequency on the reflection coefficient will be investigated.

3.4 Example

In this section, according to the model in Figure 2, the corresponding material parameters are set. By changing the delamination thickness d and the excitation frequency f of the incident wave, the variation of reflection coefficient R can be observed. The materials' properties are set as Table 1.

Firstly, the effect of delamination thickness d on reflection coefficient R is investigated. The excitation frequency of incident Stoneley wave remains unchanged, which is set as 0.5 MHz. The delamination thickness d changed from 0 to 0.02 m. The relationship between reflection coefficient R and delamination thickness d is plotted in Figure 5.

It can be seen from Figure 5 that the reflection coefficient R increases with the increase of delamination thickness d and eventually stabilizes to 0.3079 from zero. In the whole process of change, it can be roughly divided into three stages. In the first stage, the reflection coefficient

R increases from zero and the growth rate is almost unchanged. The reflection coefficient changes linearly in the first stage. In the second stage, the reflection coefficient R still increases as the delamination thickness d increases, but the growth rate gradually decreases. In the third stage, the growth rate of the reflection coefficient R gradually approached zero, and finally stabilized to 0.3079.

Secondly, the effect of frequency f on reflection coefficient R is investigated. The delamination thickness d remains unchanged, which is set as 1 mm. The excitation frequency f of incident wave changed from 0 to 10 MHz. The relationship between reflection coefficient R and frequency f is shown in Figure 6.

It can be seen from Figure 6 that the effect of excitation frequency f on the change of reflection coefficient R is the same as the effect of delamination thickness d . The change of reflection coefficient R with frequency f can be also divided into three stages, and eventually stabilizes at 0.3079. When we observe the influences of delamination thickness d and frequency f on the reflection coefficient R , geometry and frequency both affect the reflection coefficient. Since the phase velocity of Stoneley wave is constant, the change in frequency f represents the change in wavelength Λ . Therefore, the ratio of the delamination thickness d to the wavelength Λ of the incident wave can be selected as a dimensionless number to express its effect on the reflection coefficient R . In Figure 7, the variation of reflection coefficient R with the ratio d/Λ of delamination thickness to wavelength is plotted.

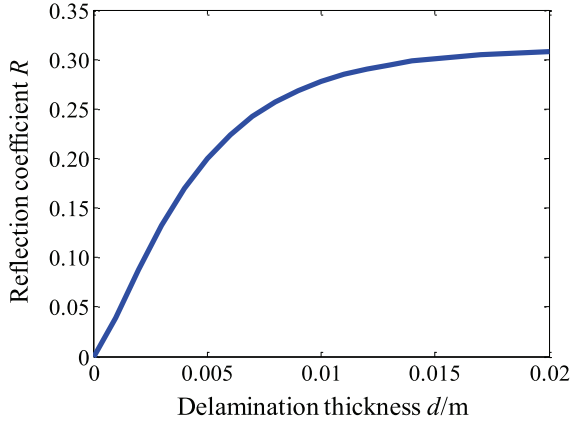
According to Figure 7, it is found that different delamination thickness follows a proportional relationship with their reflection coefficients. In two identical double-layered structures, there are two delamination with the thickness of d_1 and d_2 , respectively. At the same excitation frequency, in the stage of linear variation of reflection coefficient, the reflection coefficient of the two double-layered structures are R_1 and R_2 , respectively. According to Figure 7, the relationship between reflection coefficient and the ratio of delamination thickness to wavelength can be denoted as:

$$R = K \times \left(\frac{d}{\Lambda} \right), \quad (31)$$

where K is the proportionality coefficient. Therefore, the ratio of the two reflection coefficients R_1 and R_2 can be calculated as:

Table 1. Material property of Ω_1 and Ω_2 .

No.	ρ (kg/m ³)	E (GPa)	ν
Ω_1	2700	79	0.33
Ω_2	7850	210	0.3

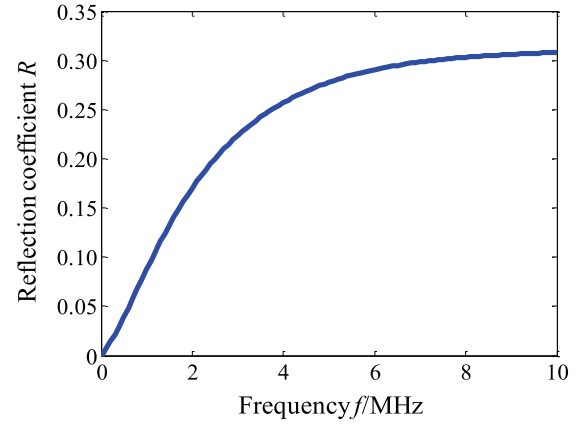
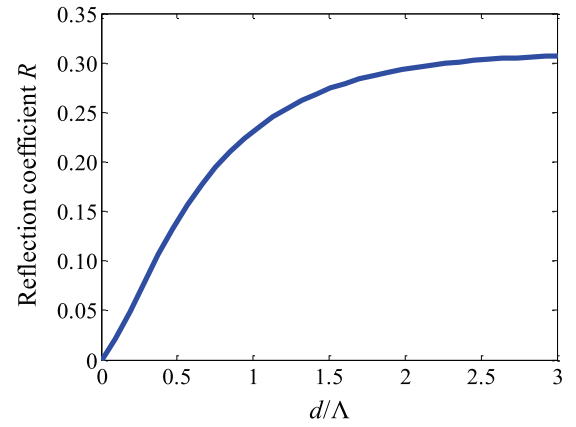
**Figure 5.** Variation of reflection R with delamination thickness d .

$$\frac{R_1}{R_2} = \frac{K \times (\frac{d_1}{\Lambda})}{K \times (\frac{d_2}{\Lambda})} = \frac{d_1}{d_2}. \quad (32)$$

In the stage of linear variation of reflection coefficient, under the same excitation frequency, the ratio of delamination thickness is equal to the ratio of corresponding reflection coefficients. The proportional relationship between the reflection coefficients and the delamination thickness can provide a reference for measuring the delamination thickness.

In addition, in Figure 7, when the abscissa is normalized to the ratio d/Λ of delamination thickness to wavelength, the variation of reflection coefficient R is consistent with Figures 5 and 6. This is because the energy distribution of Stoneley wave is related to the wavelength, and the delamination thickness can roughly reflect the blocking effect of delamination on the incident energy. Therefore, the ratio of delamination thickness to wavelength can approximately indicate the relative magnitude of the reflected energy and the total incident energy.

To briefly illustrate the relationship between reflected energy and incident energy with respect to the ratio of delamination thickness to wavelength, Figure 8 is listed. Figure 8a shows the wave structure of incident Stoneley wave in media of the material parameters in Table 1. Wave structure describes the motion amplitude of all the particles which are vertical to the direction of wave propagation corresponding one wave mode at one moment. If the incident Stoneley wave encounters the delamination with a thickness of d , as shown in Figure 2, the energy of Stoneley wave in the range of d will be reflected. In Figure 8b, the relationship between the ratio of energy in area d to total energy and the ratio of delamination thickness to wavelength is

**Figure 6.** Variation of reflection R with frequency f .**Figure 7.** Variation of reflection R with the ratio of delamination thickness to wavelength (d/Λ).

presented. The energy in area d is the energy reflected by the delamination (The energy here refers to the sum of the kinetic energy and potential energy contained in the elastic element). In Figure 8b the variation in the ratio of reflected energy to total energy is similar to the variation of the reflection coefficient shown in Figure 7. Reflected energy also starts to increase linearly, then slows down and eventually stops growing. In Figure 8b, when the ratio of delamination thickness to wavelength reaches 3, the reflected energy is close to the total energy of Stoneley wave. This explains why in Figure 7, when the ratio of delamination thickness to wavelength reaches 3, the reflection coefficient hardly increases. Therefore, through the above analysis, it can be concluded that the variation of reflection coefficient in the Figure 7 depends on the energy distribution of Stoneley wave.

Through the above analysis and calculation, the variation of reflection coefficient is obtained. At an appropriate excitation frequency such that d/Λ is low, the Stoneley wave reflection coefficient is proportional to the interfacial delamination thickness. These results can be used to establish the relationship between reflection coefficient and delamination thickness and help to measure delamination

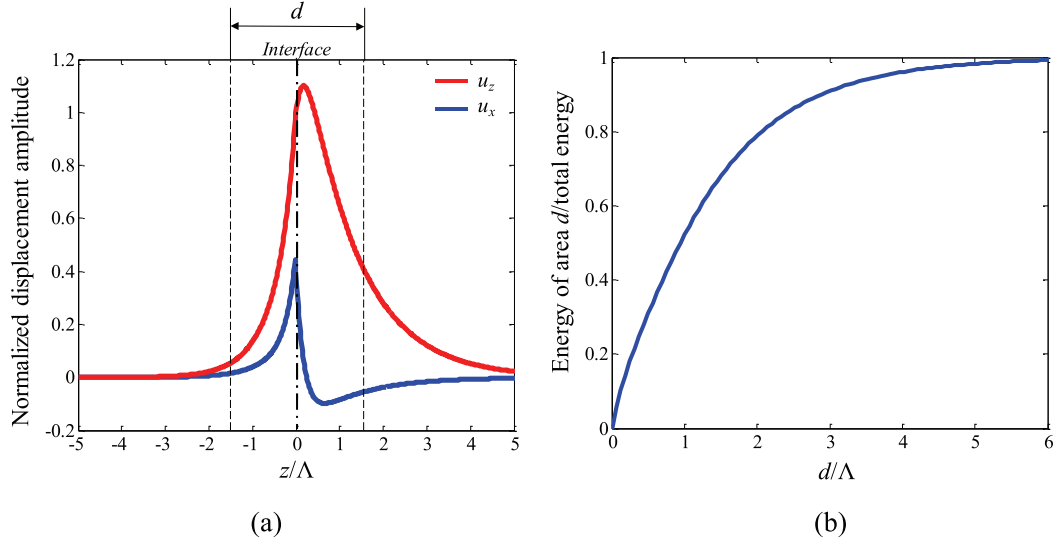


Figure 8. Wave structure and energy variation of Stoneley wave. (a) The wave structure of Stoneley wave. (b) The ratio of energy in area d to total energy.

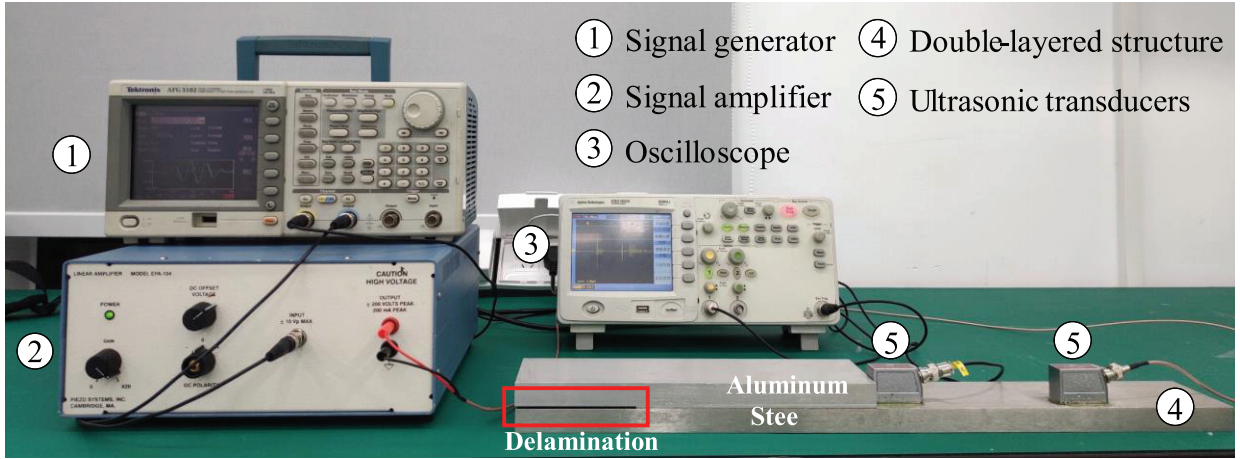


Figure 9. Experimental system.

thickness. But the results will no longer be practical when the ratio d/Λ is high because of the energy distribution of Stoneley wave.

In the next section, some experiments are carried out on two aluminum-steel double-layered structures. The relationship between reflection coefficient, delamination thickness and frequency is verified. Based on this relationship, the delamination thickness is measured.

4 Experiment validation

In order to verify the variation of reflection coefficient, an experimental system was established to measuring the reflection coefficient corresponding delamination with different thickness, as shown in Figure 9.

4.1 Experimental materials

In the experiment, we tested two double-layered structures numbered I and II with different thickness of delamination called d_1 , d_2 respectively, where $d_1 = 0.9$ mm is shown in Figure 10, d_2 is to be measured. Each double-layered structure is stacked with a steel plate and an aluminum plate, as shown in Figure 11. The material parameters, size of structure and other information are listed in Tables 2, 3 and 4.

For making the interface condition of the experimental objects meet the boundary conditions introduced in the guided wave theory, which is continuous displacement and stress, we made the following preparations. As the epoxy glue used has a hardening time of up to 1 h and a solidification time of 24 h, we applied pressure continuously

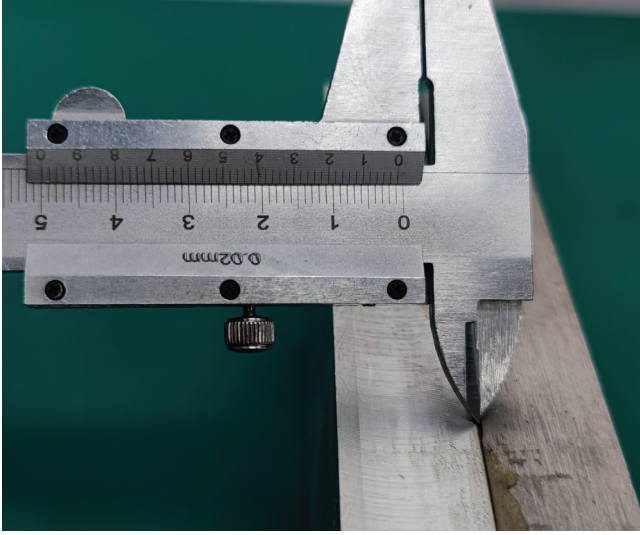


Figure 10. Measurement of delamination thickness d_1 in double-layered structure I.

to the plate within the one-hour hardening time to fully discharge the excess glue between the interfaces. In this way, the solidified glue thickness is negligible compared to

Table 4. Information of experiment equipment.

Experiment instrument	
Signal generator	Tektronix AFG3022C
Signal amplifier	Piezo EPA-104
Ultrasonic transducer	Doppler A1P13X13SW
Oscilloscope	Agilent DSO1002A
Industrial glue	Epoxy Adhesive 6240

the wavelength of the interface wave used in the experiment.

4.2 Experimental principle

A five-cycle sinusoidal wave modulated by Hanning window was set out by a signal generator (Tektronix AFG3022C) at different frequencies. After being amplified by the signal amplifier (Piezo EPA-104), the signal wave was used to drive the exciting ultrasonic transducer (Doppler A1P13X13SW) to form a Rayleigh wave packet propagating along the steel surface. As shown in Figure 12, the power of wave propagating along interface would go through the receiving point 3 times. There were three distinguishable signal wavelets detected and saved by the digital oscilloscope (Agilent DSO1002A) from the receiving

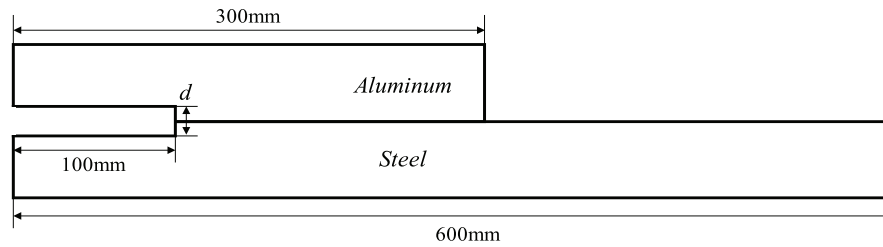


Figure 11. Diagram of double-layered structure with delamination.

Table 2. Material properties used in experiment.

Material properties					
Steel plate			Aluminum plate		
Elasticity modulus E (GPa)	Poisson's ratio ν	Density ρ (kg/m ³)	Elasticity modulus E (GPa)	Poisson's ratio ν	Density ρ (kg/m ³)
190	0.28	7900	67	0.36	2700

Table 3. Geometric parameters of double-layered structure and delamination.

Geometric parameters of double-layered structure					
Aluminum plate			Steel plate		
Length (mm)	Width (mm)	Height (mm)	Length (mm)	Width (mm)	Height (mm)
300	80	30	600	80	30
Geometric parameters of delamination					
Length (mm)			Thickness (mm)		
100			$d_1 = 0.9$ and d_2 is unknown		

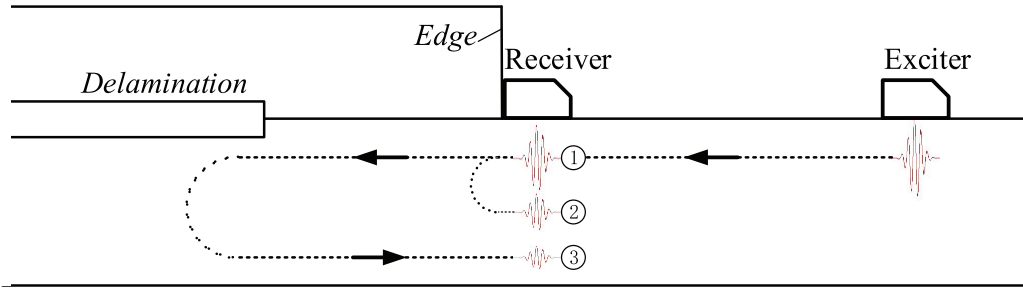


Figure 12. Measurement diagram of reflection coefficient.

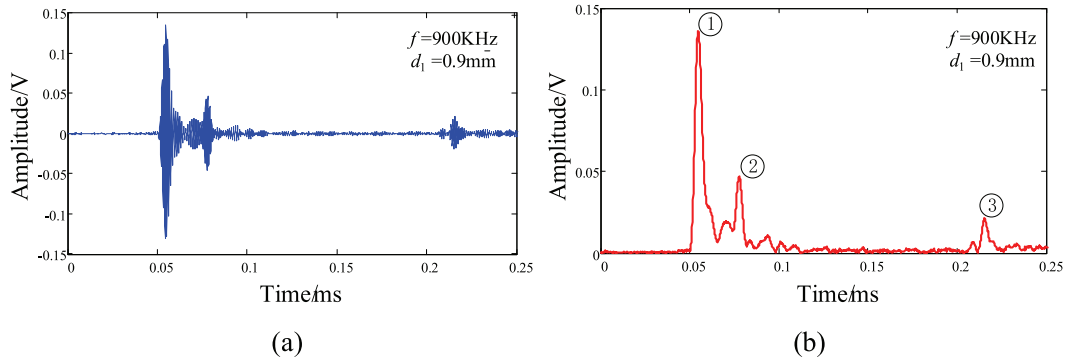


Figure 13. Experimental signal. (a) Signal processed by a band-pass filter. (b) The Hilbert transform results.

ultrasonic transducer. The first wave packet is direct Rayleigh wave, the second and third one were respectively reflected Stoneley wave from edge and boundary of delamination. We denoted amplitudes of these received wave packets as M_1 , M_2 and M_3 .

According to the above analysis, M_1 can indicate the amplitude information of the incident Stoneley wave, while M_3 can indicate the amplitude information of the reflected Stoneley wave. Therefore, the reflection coefficient R in the experiment can be calculated as:

$$R = |M_3/M_1|. \quad (33)$$

By changing the excitation frequency, the variation reflection coefficient with frequency can be measured.

4.3 Experimental results and analysis

In order to minimize the impact of the plate boundary on Stoneley wave, the excitation frequency range is selected from 0.85 MHz to 1.2 MHz and measured three times per 25 KHz. The average value of the three signals is then used as the experimental result to calculate the reflection coefficient. Some signals are shown in Figure 13.

It should be note that the signals are processed by a band-pass filter to remove some interference of low and high frequency components. The bandwidth of the band-pass filter is 0.2–3 MHz. By extracting the corresponding amplitude of the wave packet, the variation of the reflection coefficient with the excitation frequency can be obtained.

For the double-layered structure I with delamination thickness of $d_1 = 0.9$ mm, the variation of measured reflection coefficient R_1 is presented in Figure 14.

For the double-layered structure II with delamination thickness of d_2 , the variation of measured reflection coefficient R_2 is presented in Figure 15

According to Figures 14 and 15, it can be seen that in the two double-layered structures with different delamination thickness, the reflection coefficient basically changes linearly with excitation frequency. This experimental conclusion is consistent with the theoretical prediction and verifies the correctness of the theoretical prediction.

4.4 Measurement of delamination thickness

In the experiment, it is difficult to accurately measure the reflection coefficient caused by delamination due to the limitation of the ultrasonic transducer efficiency. In the experiment, the correspondence between reflection coefficient and delamination thickness in Figure 7 cannot be directly used to determine the delamination thickness by measuring reflection coefficient. In general, the relative magnitude or variation trend of the reflection coefficient can be measured accurately in experiments. Therefore, the delamination thickness can be measured by the principle that the ratio of reflection coefficient is equal to the ratio of the delamination thickness shown in equation (32). This method can avoid the influence of ultrasonic transducer's efficiency on the measured results.

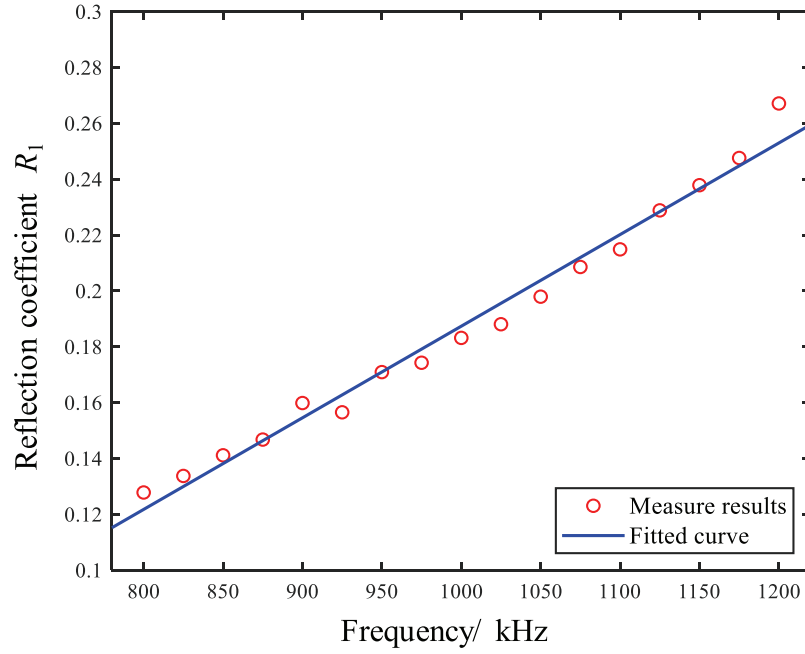


Figure 14. Measured reflection coefficient with delamination thickness of d_1 .

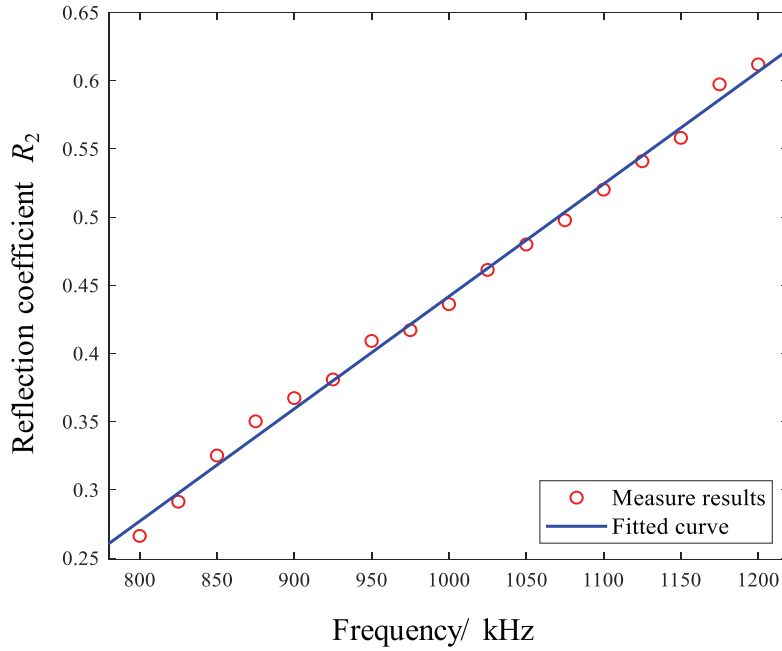


Figure 15. Measured reflection coefficient with delamination thickness of d_2 .

According to equation (32), under the same excitation frequency, the ratio of the measured reflection coefficient and the ratio of the delamination thickness in these two experimental structures should be equal, as $R_2/R_1 = d_2/d_1$. R_1 , R_2 and d_1 are all measured, so d_2 can be determined.

The ratio of reflection coefficients (R_2/R_1) at each frequency is calculated and the average value is 2.379 shown

in Figure 16. According to equation (32), the delamination thickness d_2 in double-layered structure II is 2.14 mm. Measured by vernier caliper, the actual delamination thickness d_2 is 2.1 mm. The relative error between the predicted value in experiment and the actual measured value is 1.9%. The experimental predicted value is in good agreement with the actual delamination thickness. The experimental results can prove the proportional relationship between the

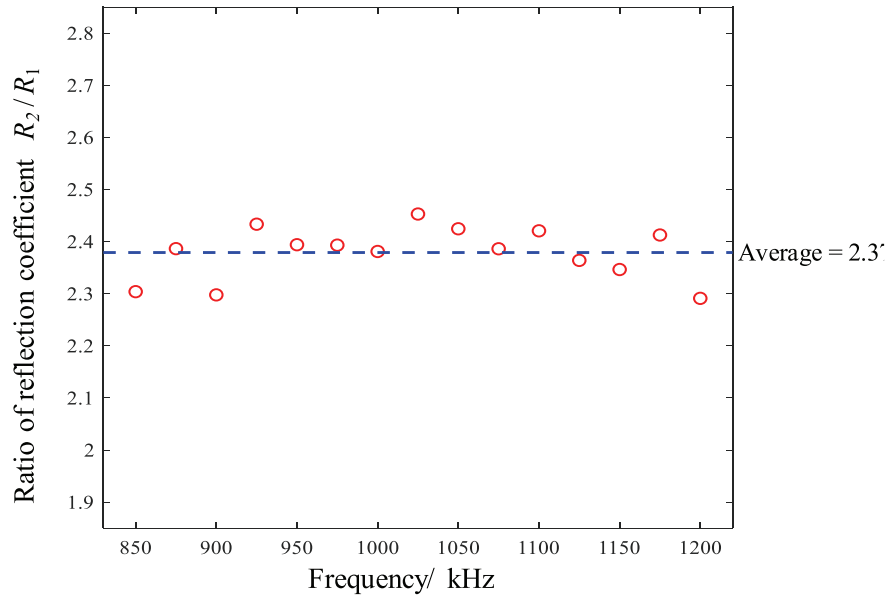


Figure 16. Ratio of measured reflection coefficient.

reflection coefficients and the delamination thickness and provides a reference for the measurement of delamination thickness.

5 Conclusion

In this paper, the interaction between Stoneley wave and delamination in a double-layered structure is studied by reciprocity theorem. The influences of delamination thickness and excitation frequency on reflection coefficient are investigated. In experiments, the variation of reflection coefficient is verified and the delamination thickness is measured by reflection coefficient.

In the first part, the relationship between reflection coefficient, delamination thickness and frequency is calculated via reciprocity theorem. In order to discuss the variation of reflection coefficient in detail, an example is given. Firstly, the influences of excitation frequency f and delamination thickness d on the reflection coefficient are analyzed, respectively. It is found that the excitation frequency f and the delamination thickness d have the same effect on the reflection coefficient. The reflection coefficient increases linearly at first, then the rate of increase slows down, and finally tends to be stable. Secondly, according to the influences of excitation frequency f and delamination thickness d on the reflection coefficient, these two physical quantities are normalized into the ratio of delamination thickness to wavelength (d/Λ). It is found, in the stage of linear variation of reflection coefficient, under the same excitation frequency such that d/Λ is low, the ratio of delamination thickness is equal to the ratio of corresponding reflection coefficients. In order to explain the variation of the reflection coefficient, the energy distribution of Stoneley is

investigated. It is found that the energy distribution is similar to that of the reflection coefficient. Therefore, the ratio of delamination thickness to wavelength (d/Λ) can approximately indicate the relative magnitude of the reflected energy and the total incident energy, and the variation of reflection coefficient is determined by the energy distribution of Stoneley wave.

In the second part, some experimental validations are carried out on two aluminum-steel plates I and II. There is a delamination in these double-layered structures I and II, and the delamination thickness is d_1 and d_2 . d_1 in double-layered structure I is 0.9 mm and d_2 in double-layered structure II is unknown. The reflection coefficients of two aluminum-steel plates with delamination thickness of d_1 and d_2 are measured, respectively. It is found that the reflection coefficients in double-layered structures I and II both increase linearly with the excitation frequency. In addition, according to the theoretical relationship between reflection coefficient, delamination thickness and frequency the delamination thickness d_2 is calculated as 2.14 mm. The actual value of the delamination thickness d_2 is 2.1 mm, and the relative error is 1.9%. The experimental results can verify the theoretical relationship between the reflection coefficients and the delamination thickness. The delamination thickness d_2 is measured in a small error.

The conclusion above lay a theoretical foundation for delamination thickness measurement in composite structures in Stoneley-wave-based non-destructive testing. However, the theoretical model of this paper is a simplified two-dimensional model. The actual interfacial damages are three-dimensional and usually own more complex shapes. It requires further research to effectively detect and reconstruct interfacial damages of composite structures.

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Conflict of interest

The authors declare that they have no conflicts of interest in relation to this article.

Data availability statement

The research data associated with this article are included within the article.

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